

Proper Scaling of Anomalous Hall Effect

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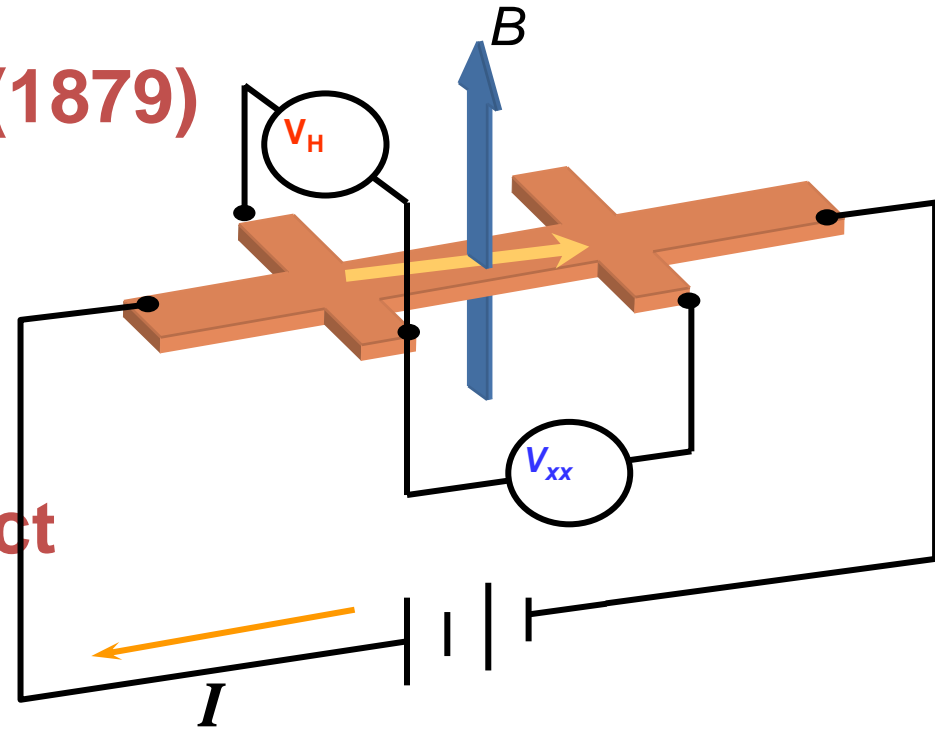
Hall effect

- Ordinary Hall effect (1879)

$$\rho_{xy} = R_0 B$$

- Anomalous Hall effect (1880&1881)

$$\rho_{xy} = R_0 B + R_{ah} M_z$$



$$\rho_{xy} = R_0 B + \rho_{ah} M_z$$

$$\rho_{ah} = f(\rho_{xx})$$

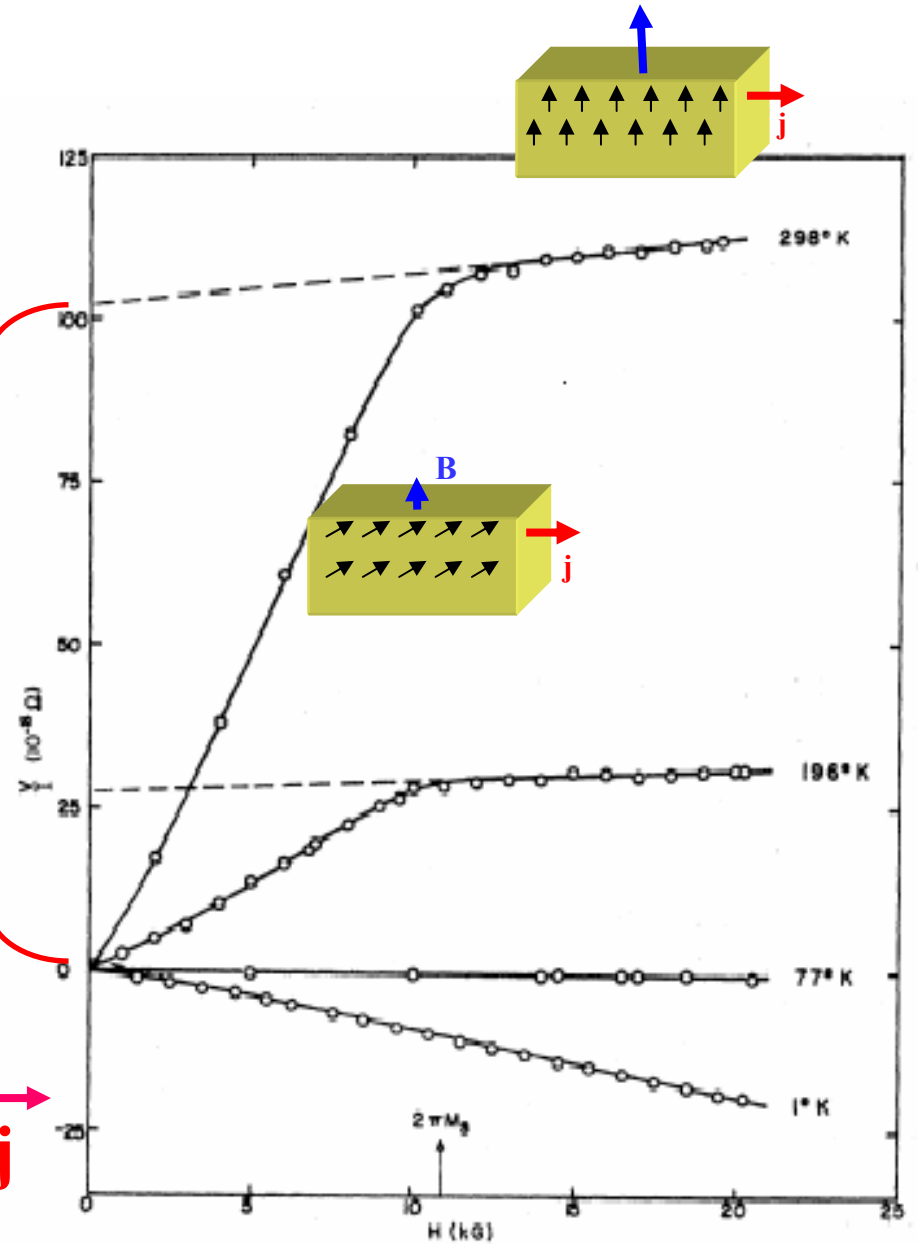
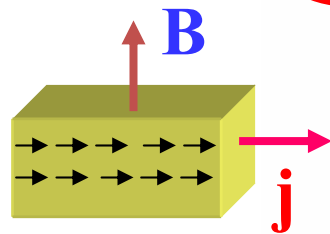
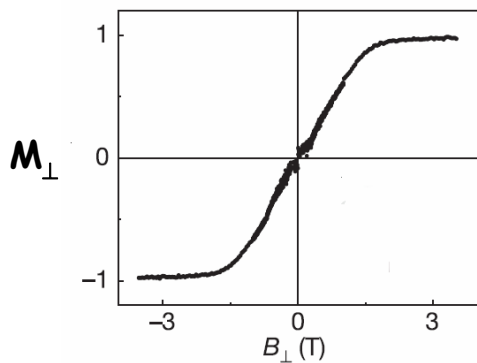
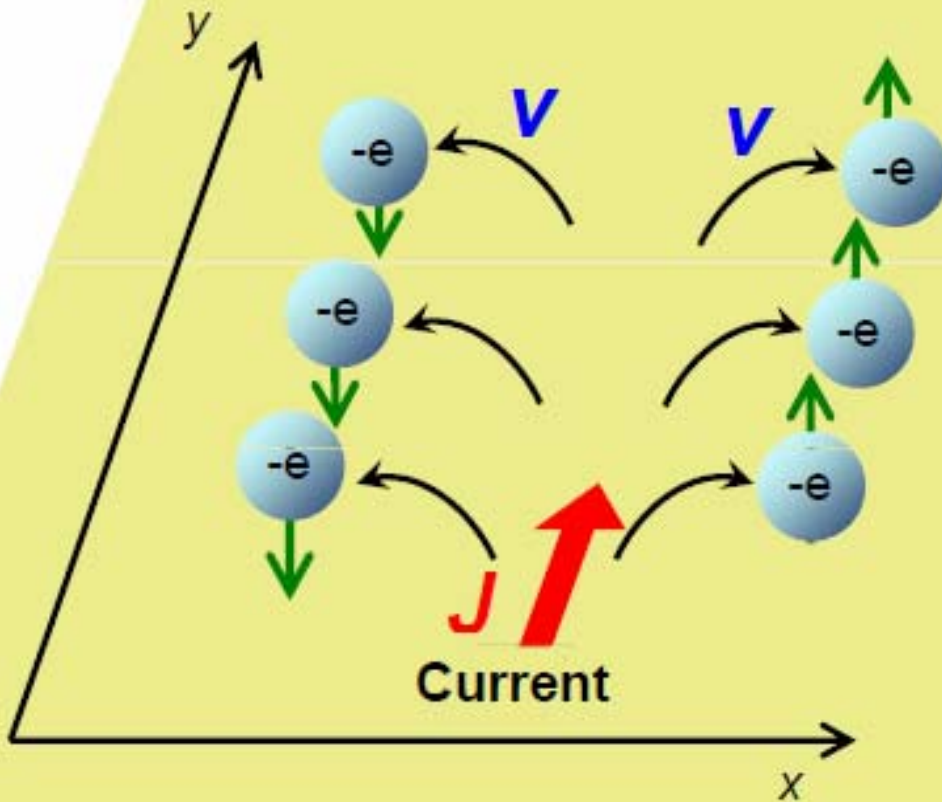


FIG. 1. Variation of the Hall resistance with applied magnetic field for whisker Fe

Spin Hall Effect



Theories

- **Karplus & Luttinger (1954)**

Intrinsic: spin-orbit interaction with nonzero interband matrix element of the current.

Anomalous Velocity

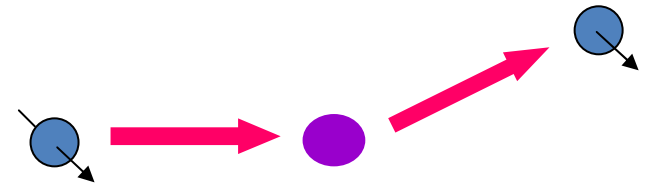


$$\rho_{in} \propto \rho_{xx}^2$$

- **J. Smit (1955)**

Extrinsic: spin-orbit interaction with asymmetry scatterings at impurities.

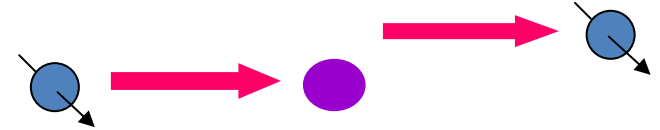
Skew Scattering



$$\rho_{sk} \propto \rho_{xx}$$

Theories

- **L. Berger (1970)** Side Jump



Extrinsic: spin-orbit interaction with side-jump scatterings at impurities.

$$\rho_{sj} \propto \rho_{xx}^2$$

$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

G. Sundaram and Q. Niu, Phys. Rev. B 59 (1999) 14915.

$$\frac{d \vec{r}(t)}{dt} = \frac{\partial \varepsilon_n(\vec{k})}{\partial \vec{k}} - \vec{B}_n(\vec{k}) \times \frac{d \vec{k}(t)}{dt}$$

k- space curvature

$$\frac{d \vec{k}(t)}{dt} = \frac{\partial V(\vec{r})}{\partial \vec{r}} - \vec{B}(\vec{r}) \times \frac{d \vec{r}(t)}{dt}$$

r- space curvature

Theories

Karplus-Luttinger Term — Berry Curvature

T. Jungwirth, Q. Niu and A. H. MacDonald: Phys. Rev. Lett. 88 (2002) 207208.
M. Onoda and N. Nagaosa: J. Phys. Soc. Jpn. 71 (2002) 19.

Berry curvature

$$\mathbf{j}_y = -e^2 \mathbf{E} \times \int d^3k f(\mathbf{k}) \underbrace{\Omega}_{\propto \sigma_{int}}$$

$$\sigma_{int} = \text{constant}$$



$$\rho_{int} = \sigma_{int} \rho_{xx}^2$$

Extrinsic or Intrinsic ???

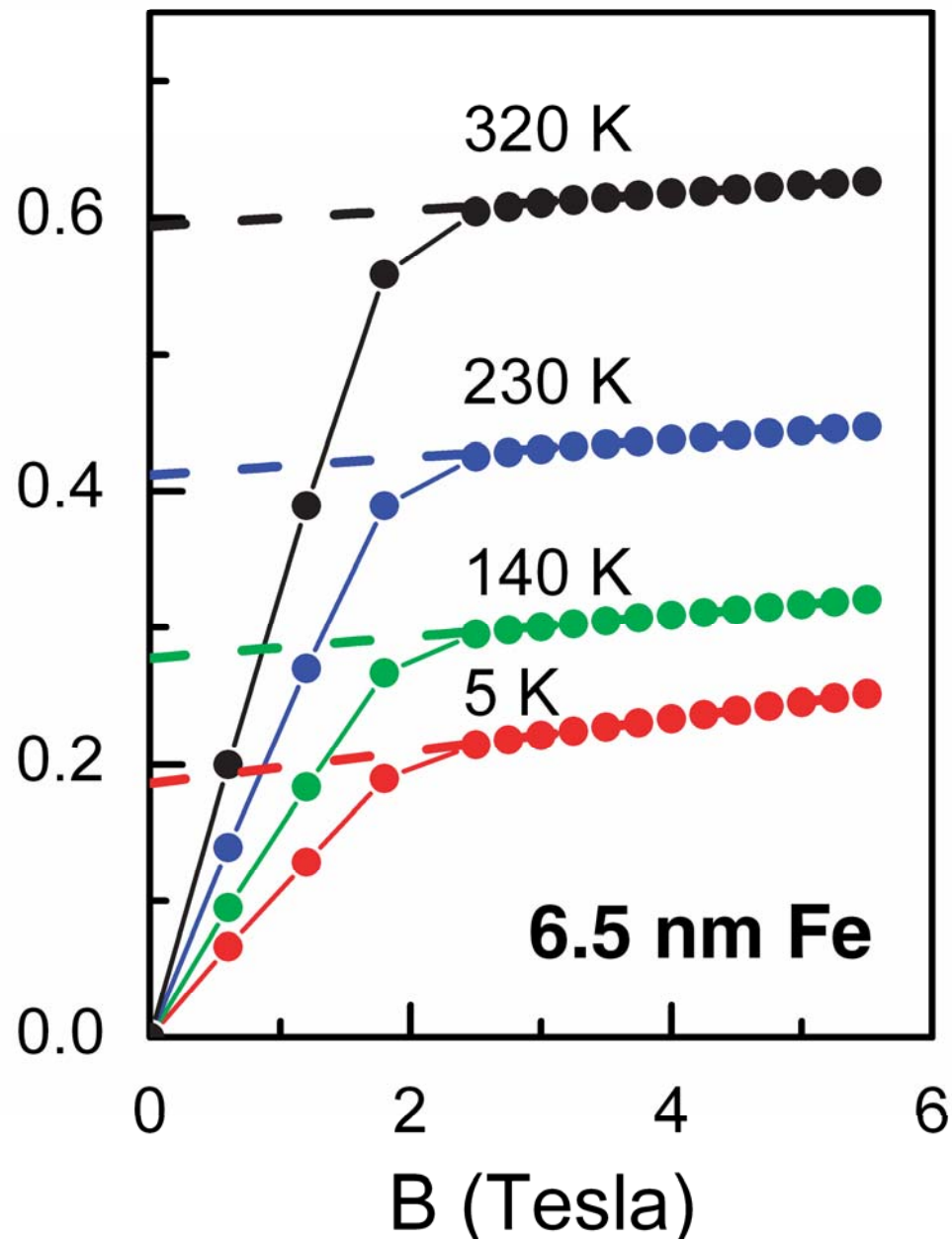
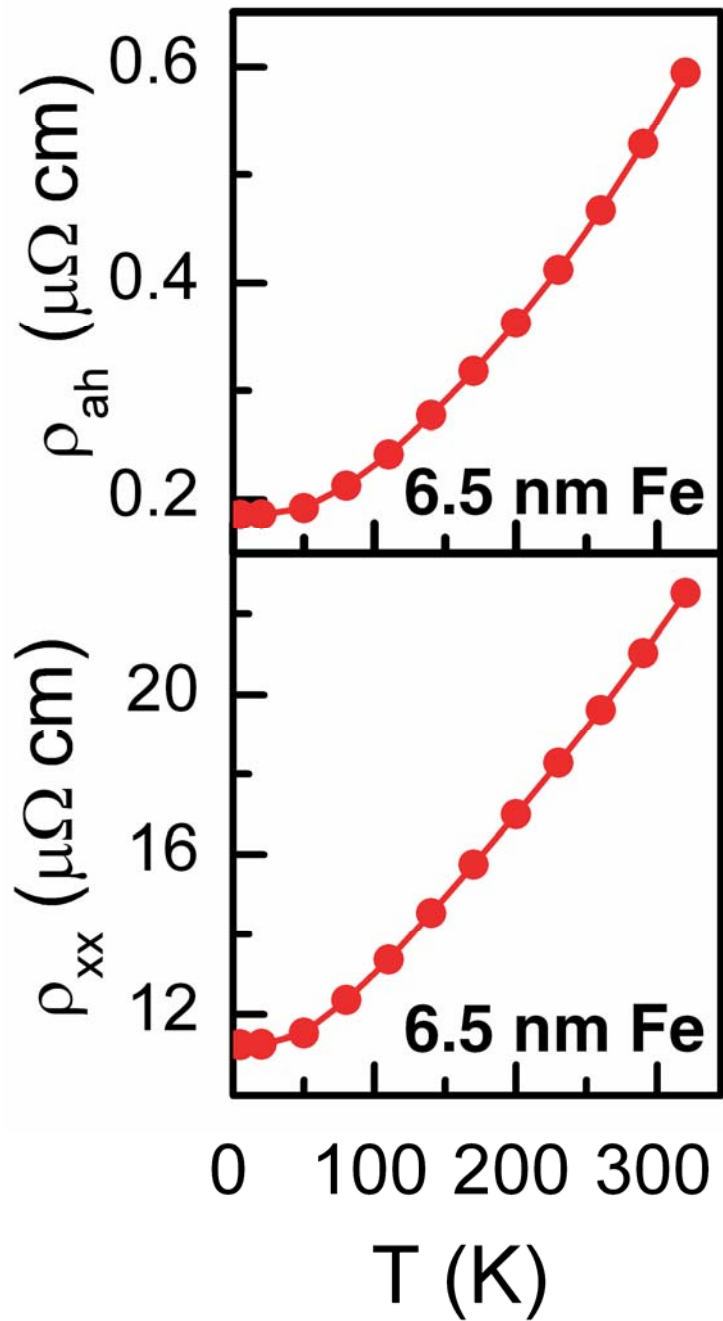


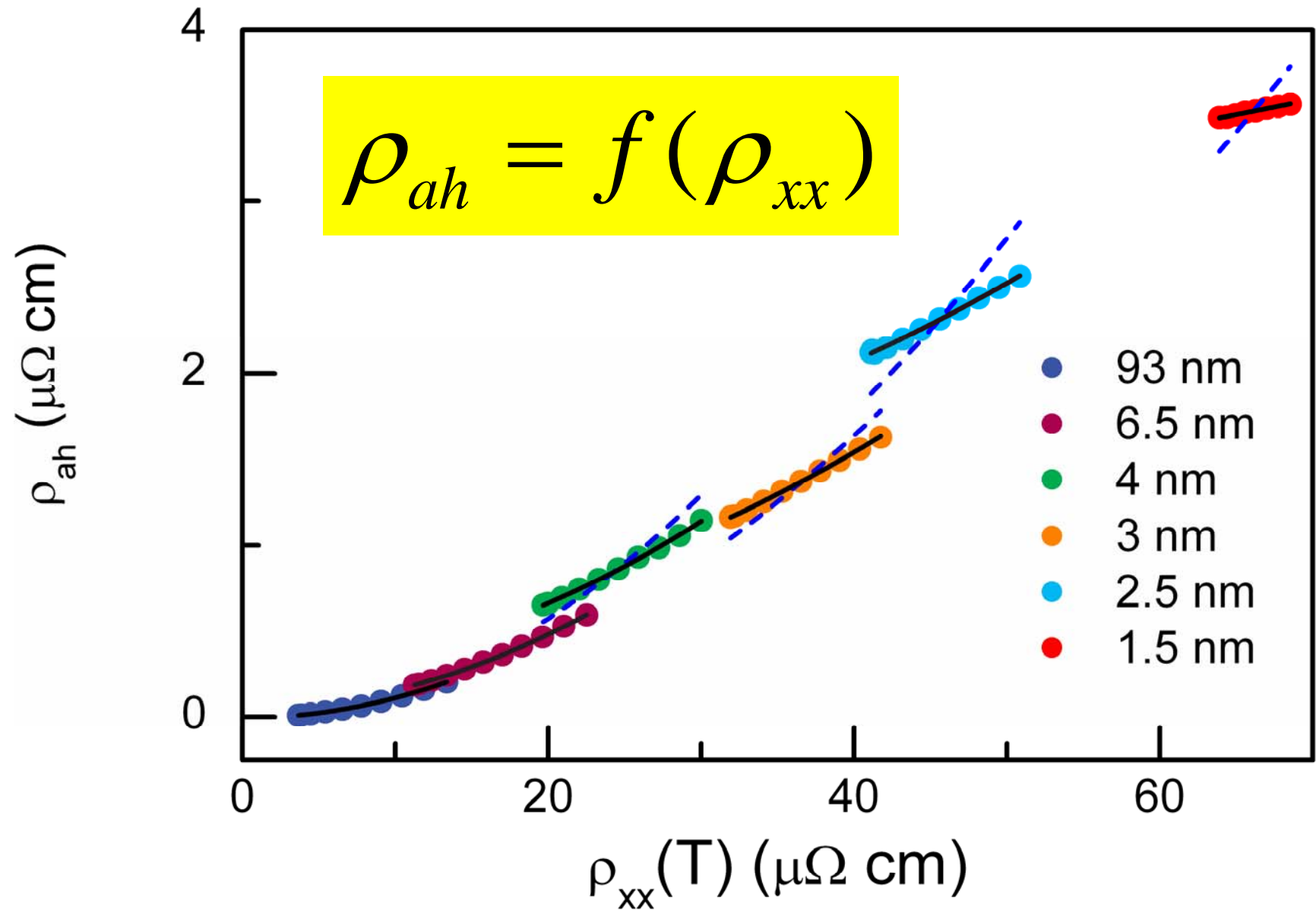
$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

Question 1:
Proper Scaling of the AHE ?

$$\rho_{ah} = f(\rho_{xx})$$

Question 2:
Intrinsic and Extrinsic in the AHE ?





Can we use $\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$?

Matthiessen's rule :

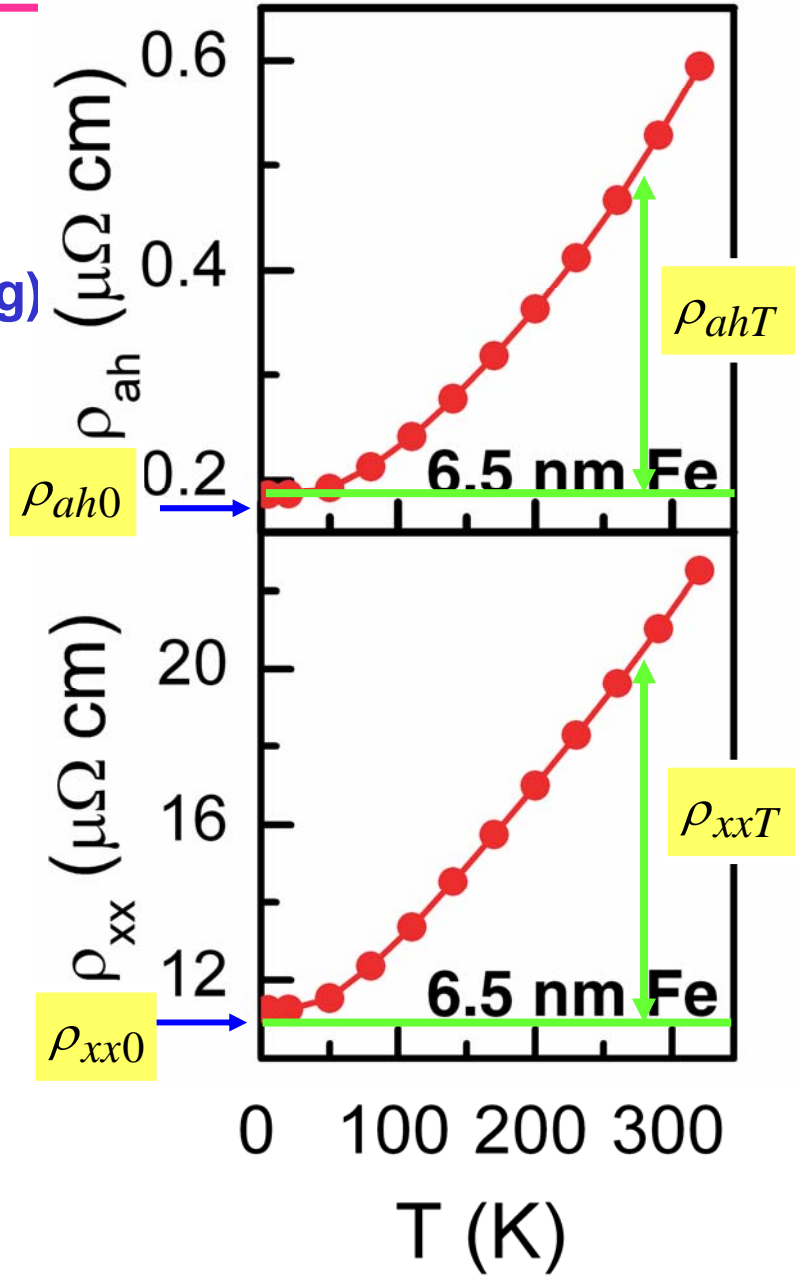
$\rho_{xx0} \sim$ Impurity (Skew scattering)

$$\rho_{xx} = +$$

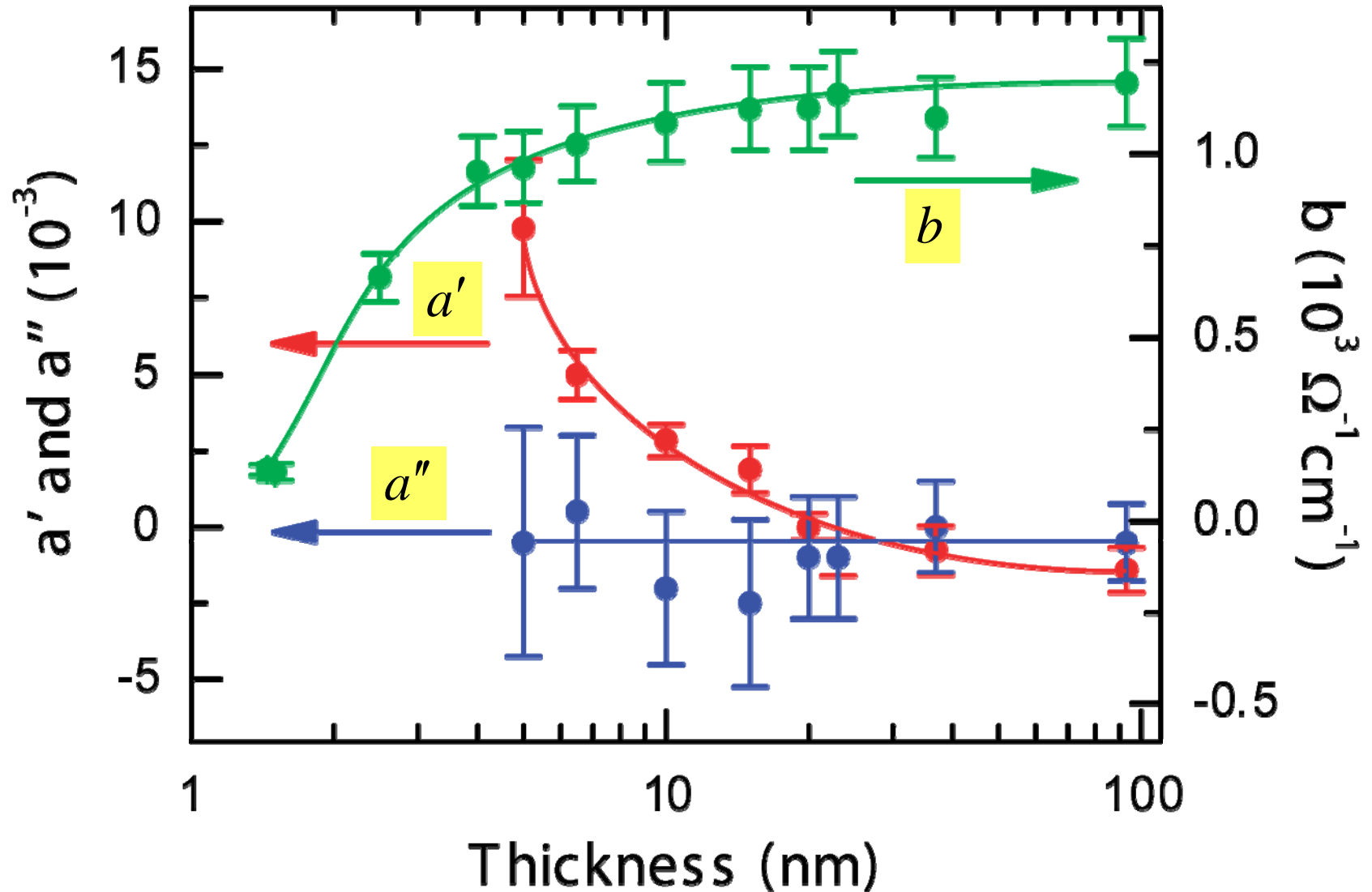
$\rho_T \sim$ Phonon (Unknown)

$$a\rho_{xx} = a\rho_{xx0} + a\rho_T$$

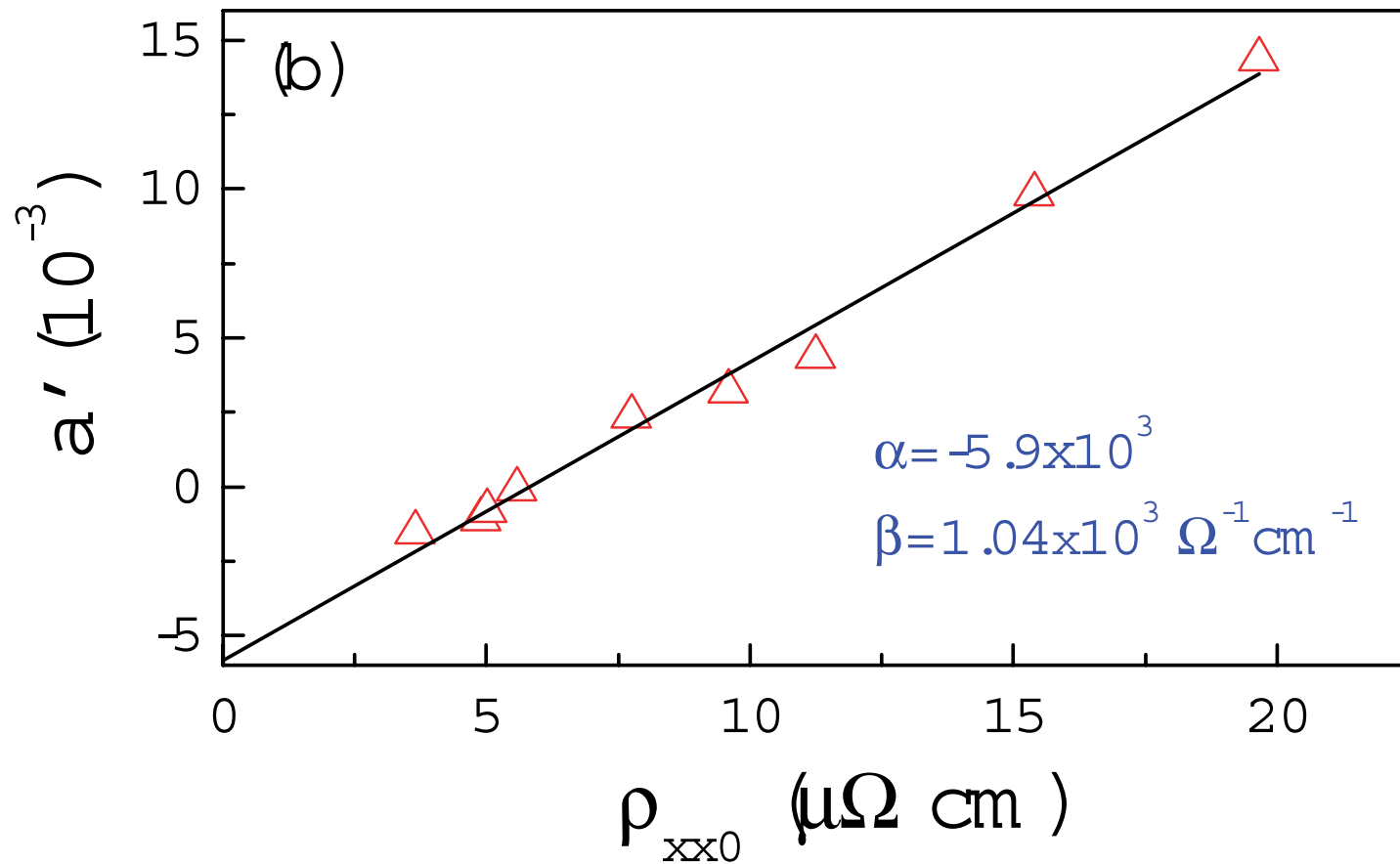
$$\longrightarrow a'\rho_{xx0} + a''\rho_T$$



$$\rho_{ah} = a' \rho_{xx0} + a'' \rho_{xxT} + b(\rho_{xx0} + \rho_{xxT})^2$$



$$a' = \alpha + \beta \rho_{xx0}$$



Old scaling:

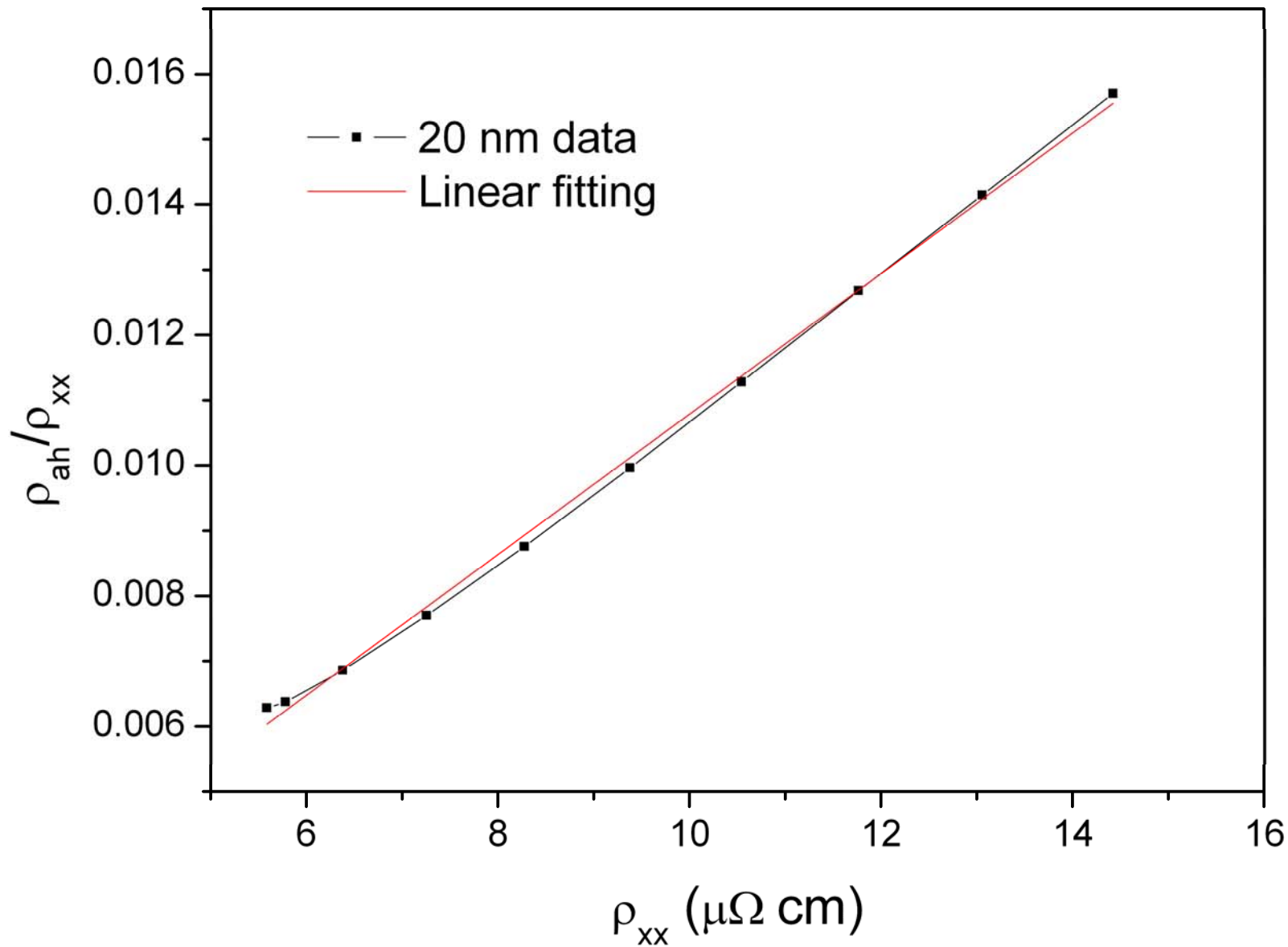
$$\rho_{ah} = a\rho_{xx} + b\rho_{xx}^2$$

$$\frac{\rho_{ah}}{\rho_{xx}} = a + b\rho_{xx}$$

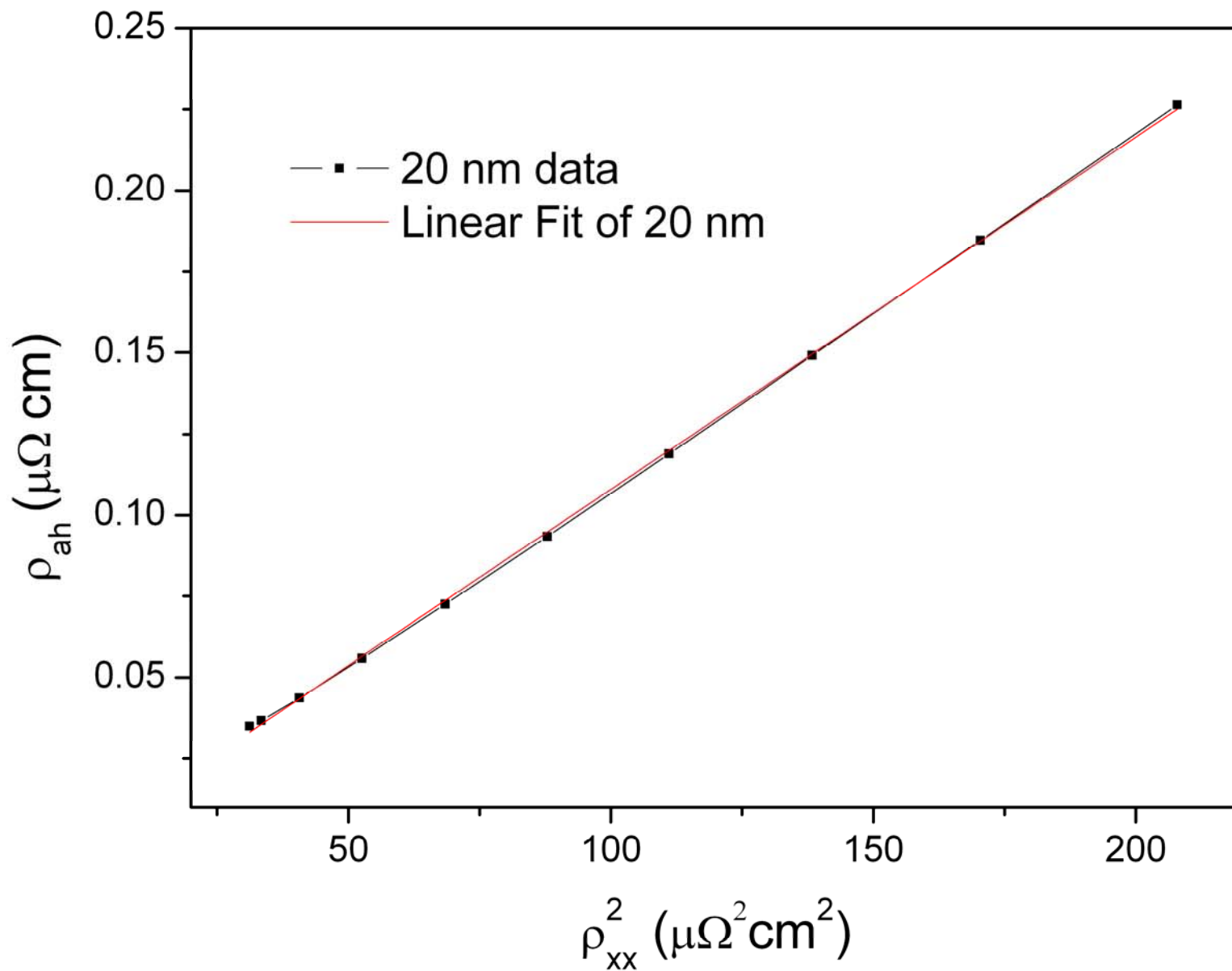
New scaling:

$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + b\rho_{xx}^2$$

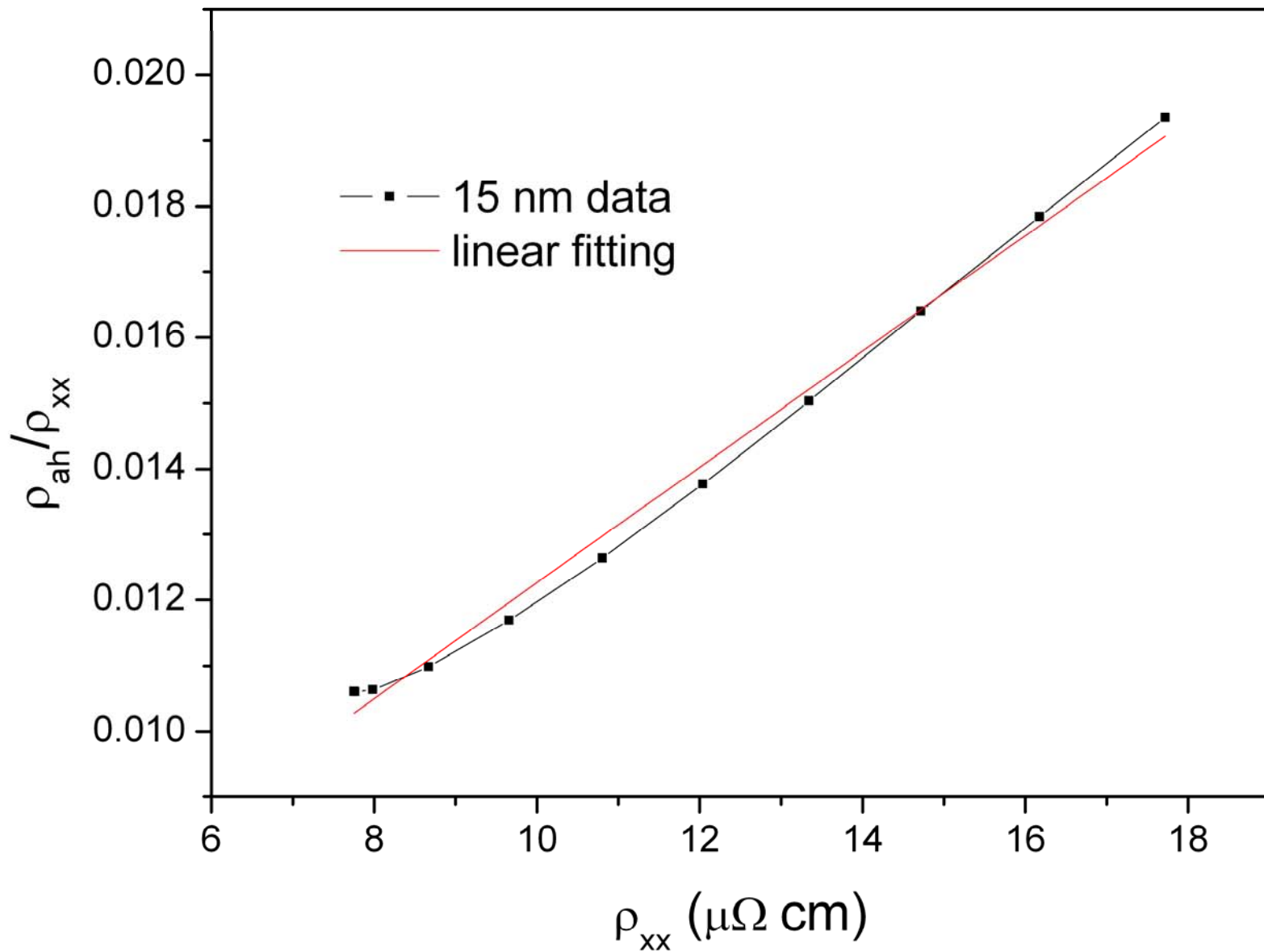
Old scaling:



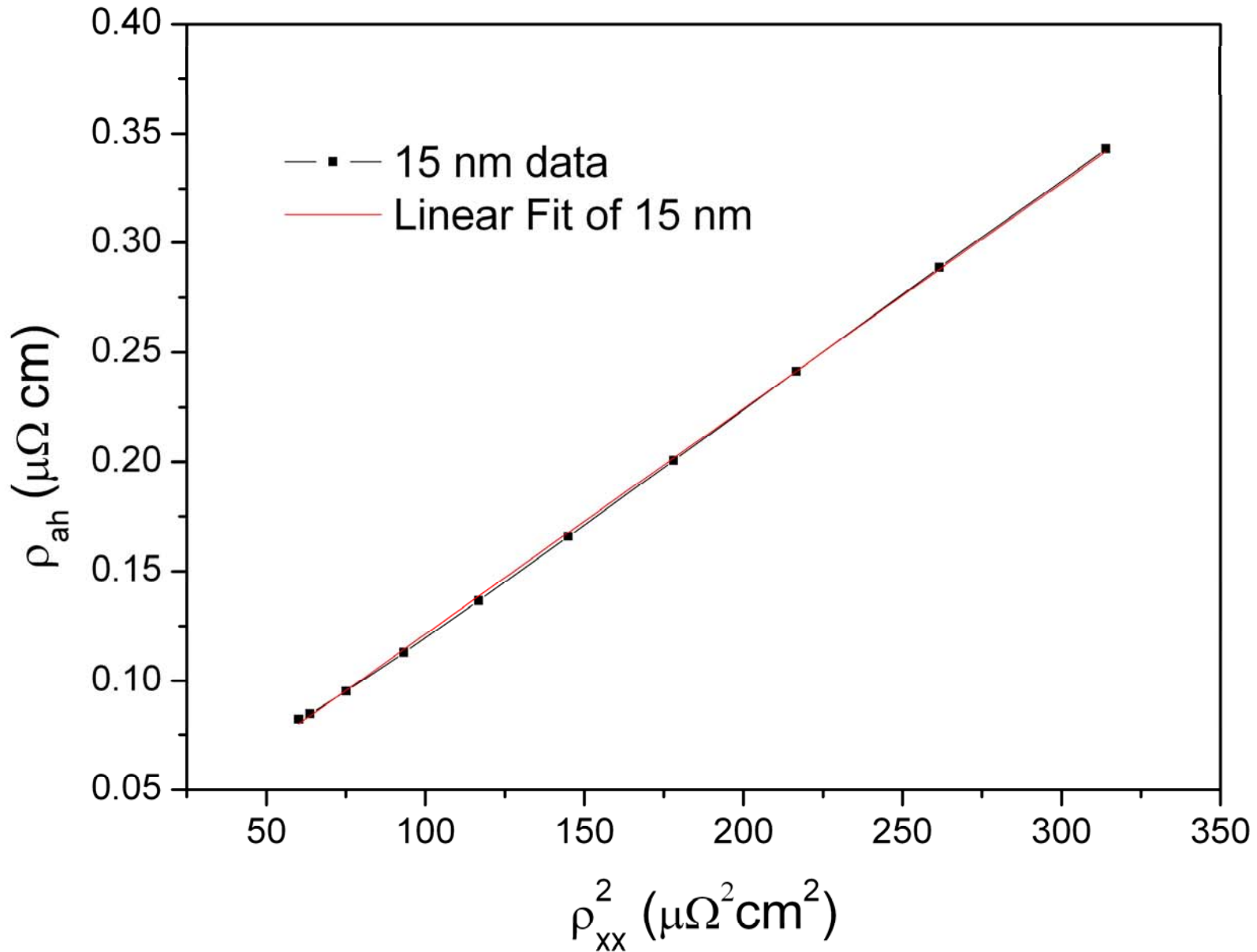
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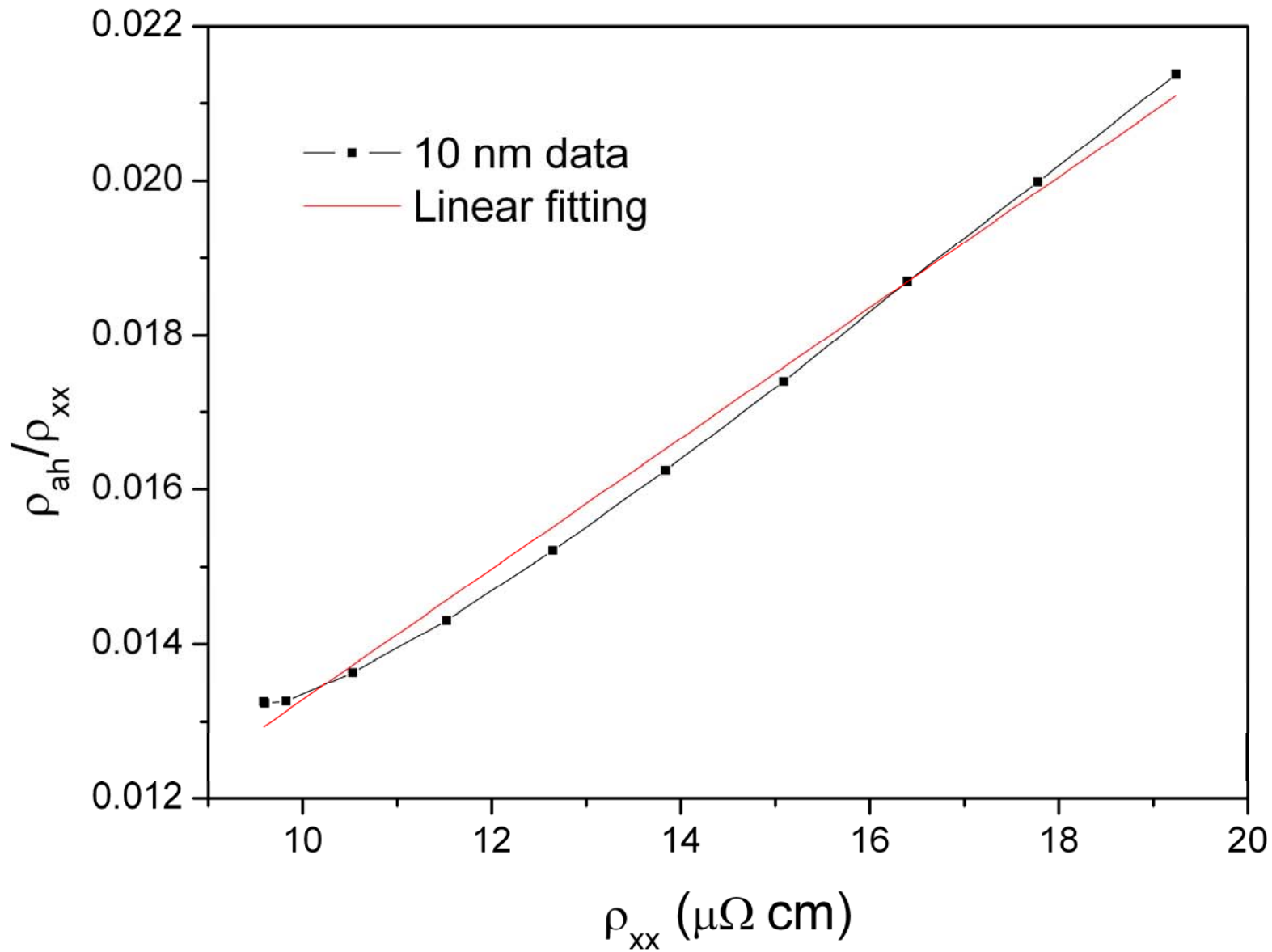
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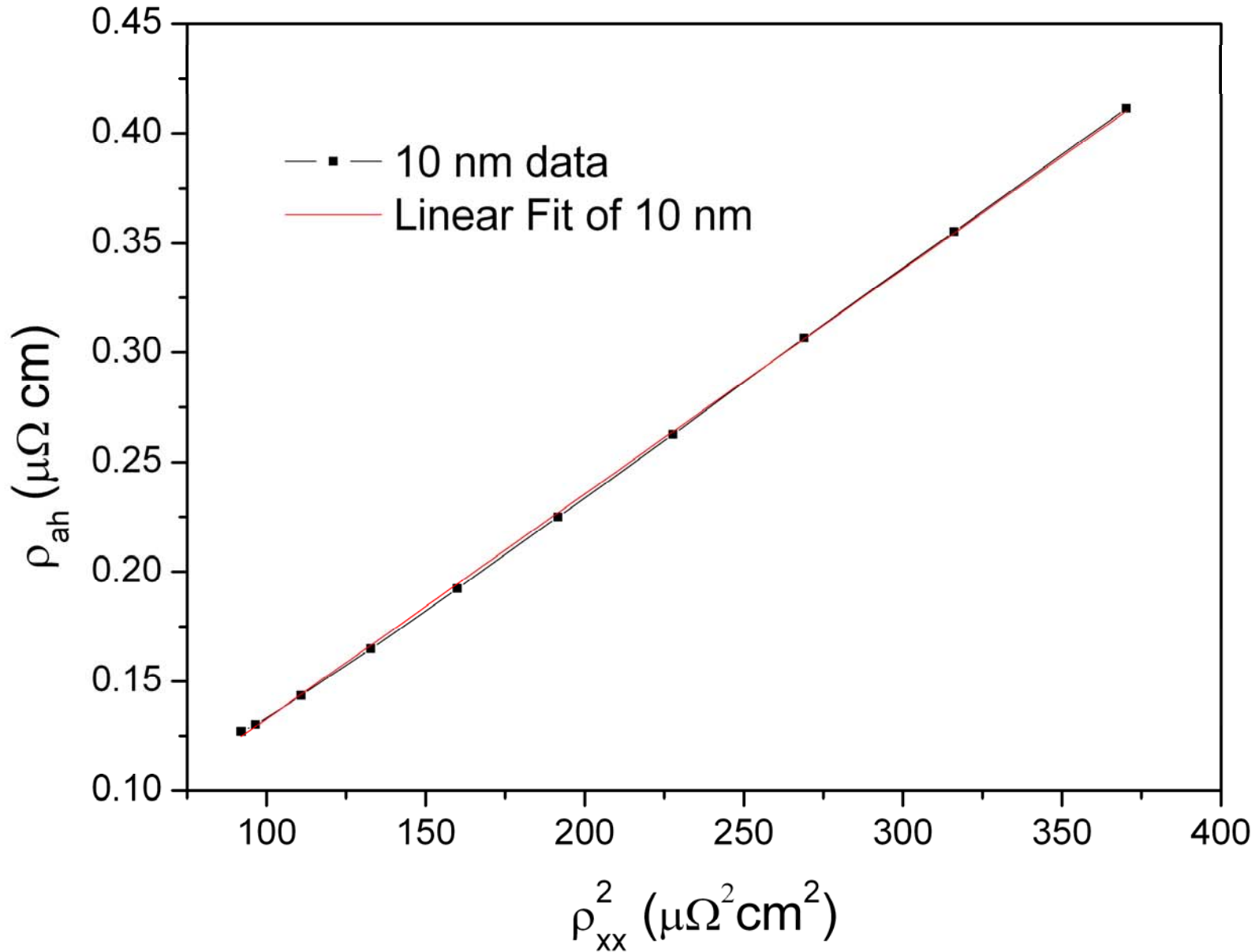
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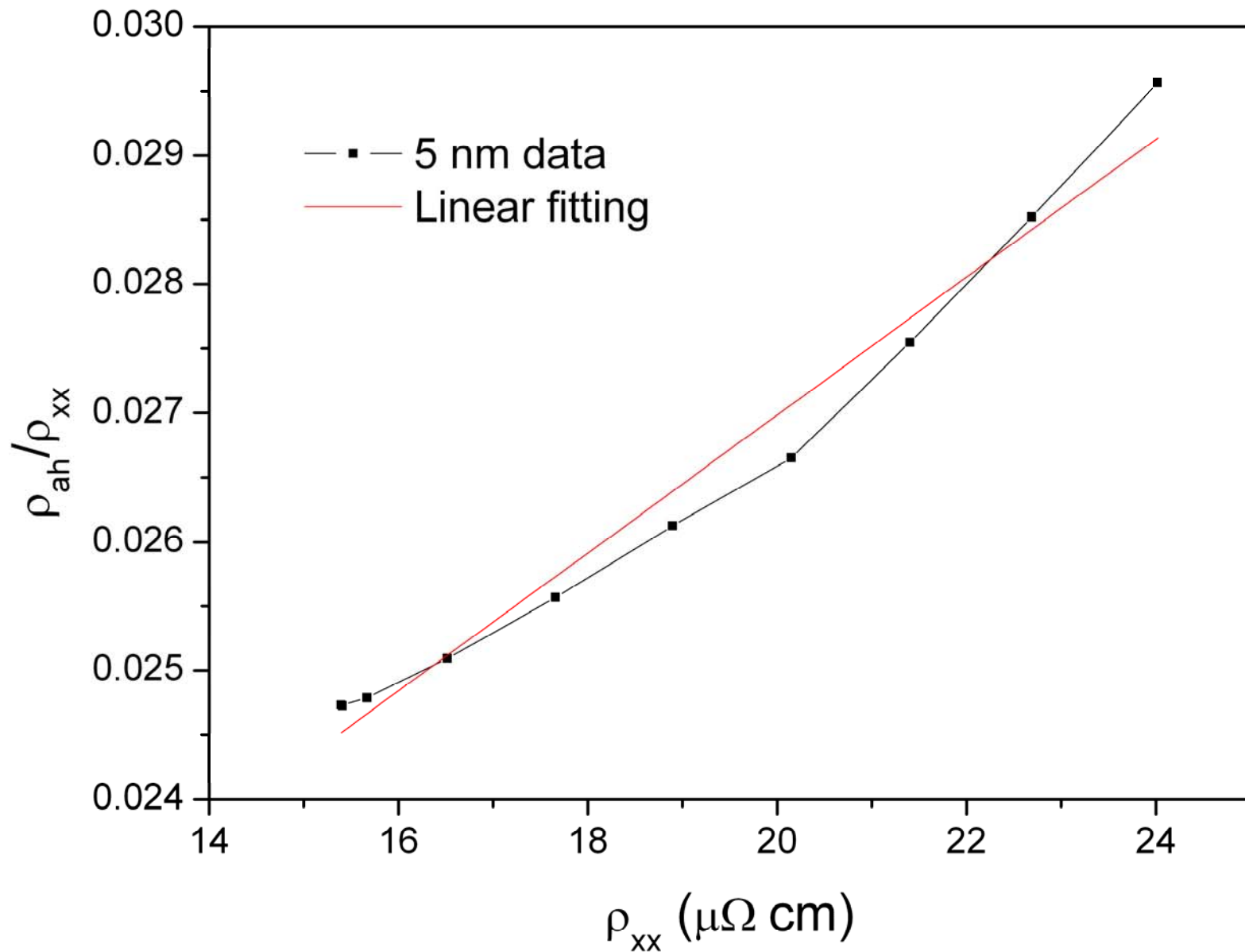
Old scaling:



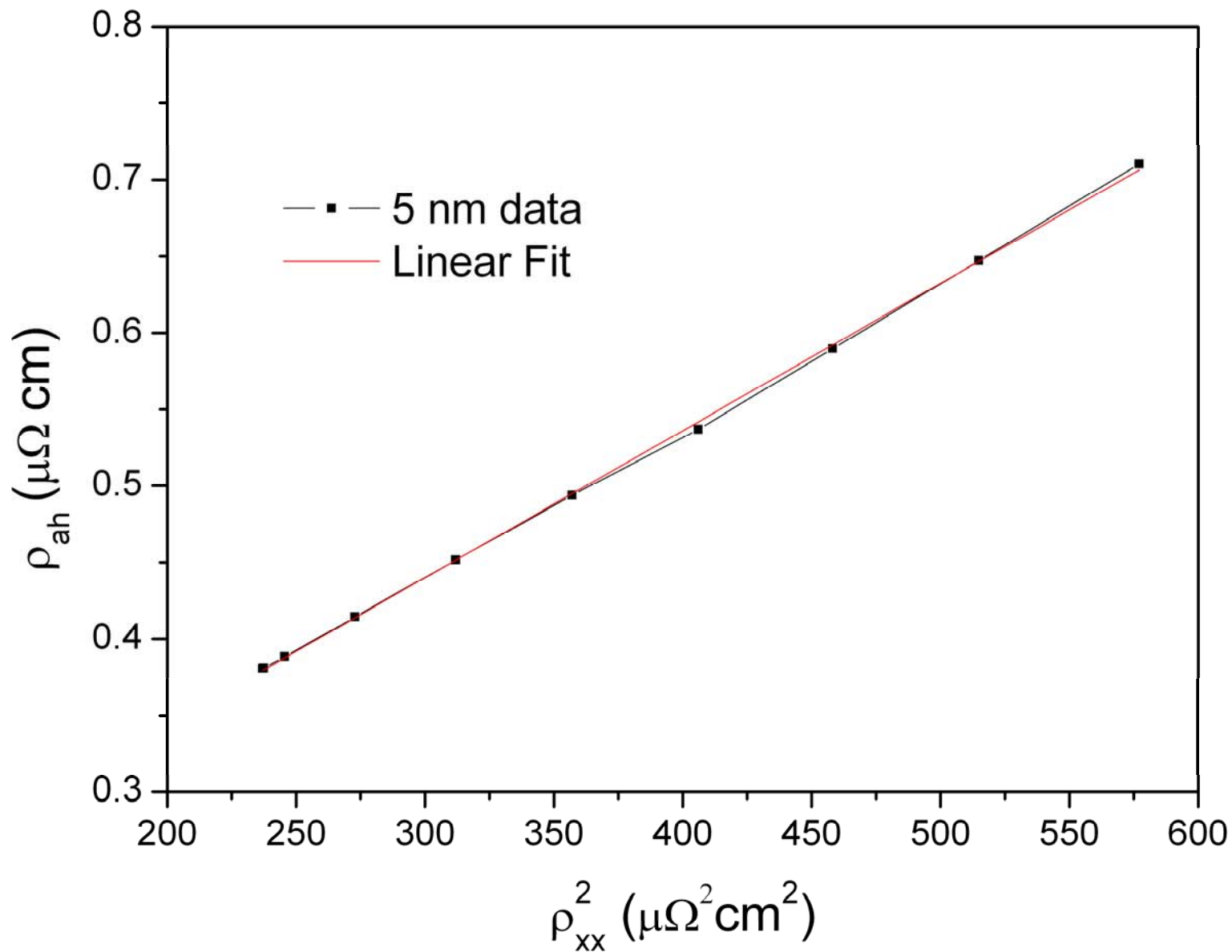
New scaling:



Old scaling:



New scaling:



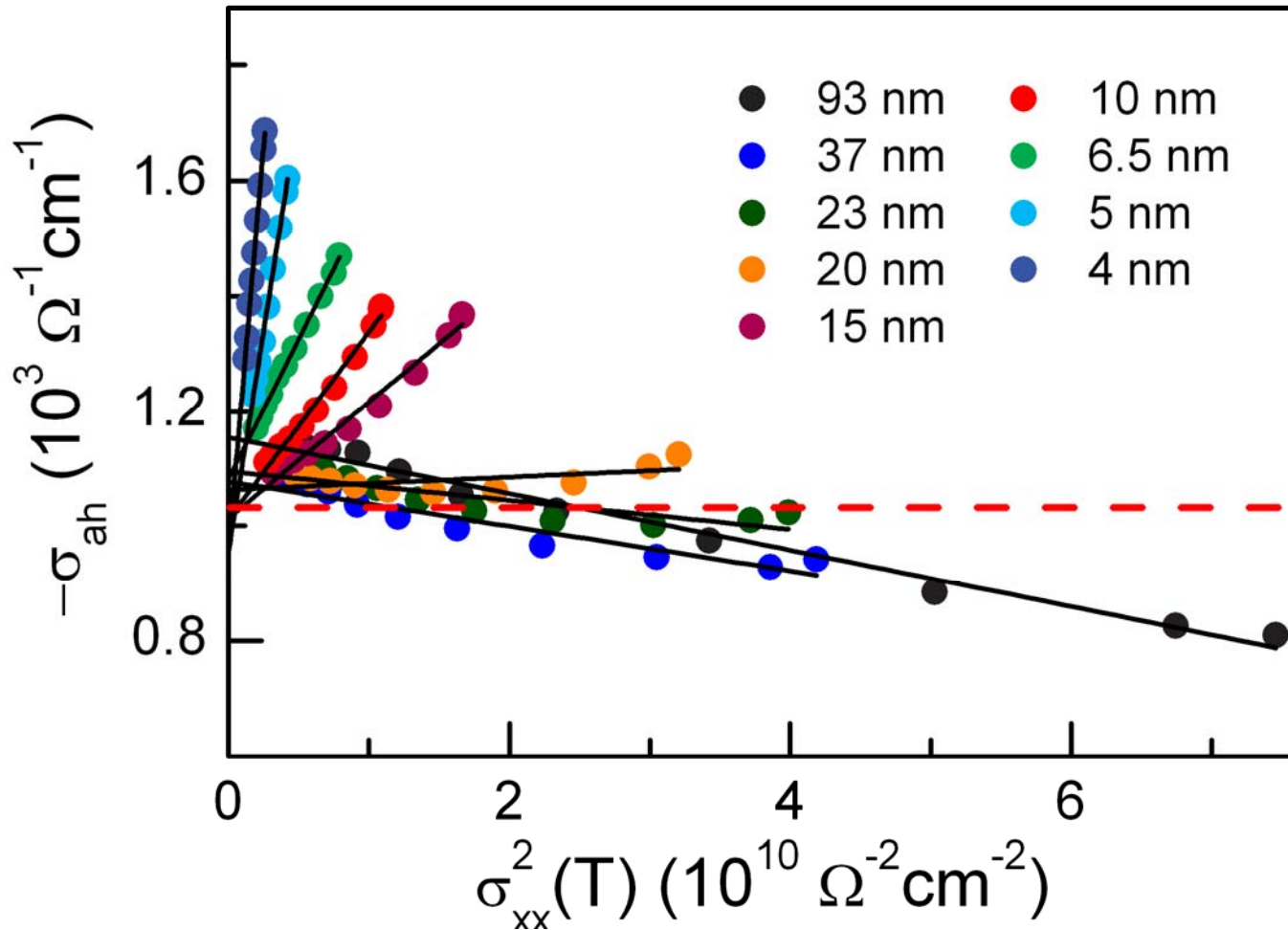
Proper Scaling of the AHE

$$\rho_{ah} = f(\rho_{xx0}, \rho_{xx}) = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + b\rho_{xx}^2$$

$$\sigma_{ah} = \sigma(\sigma_{xx0}, \sigma_{xx}) = -(\alpha\sigma_{xx0}^{-1} + \beta\sigma_{xx0}^{-2})\sigma_{xx}^2 - b$$

What β and b really mean ???

$$\sigma_{ah} = -(\alpha\sigma_{xx0}^{-1} + \beta\sigma_{xx0}^{-2})\sigma_{xx}^2 - b$$

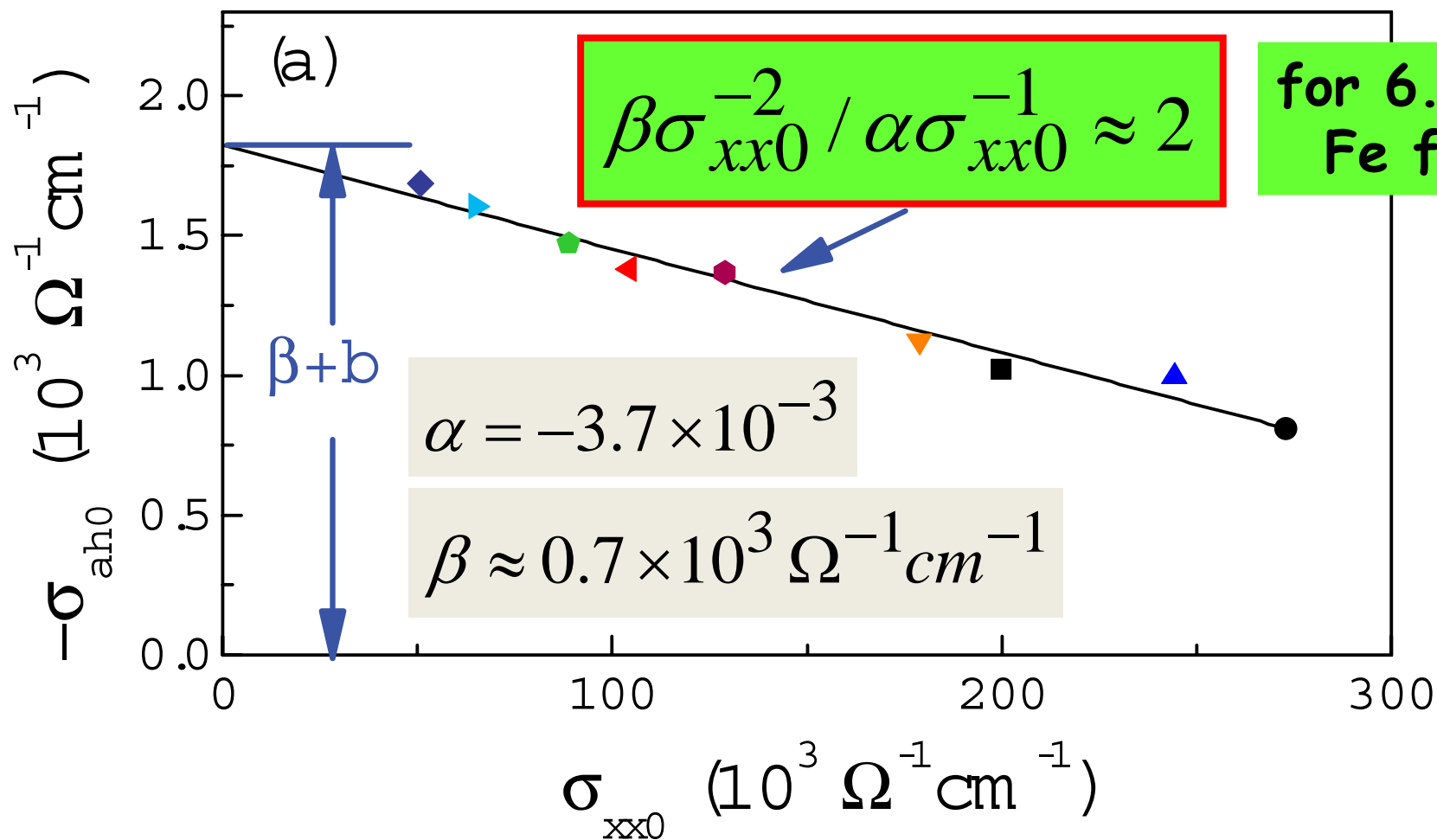


$$b \approx 1.1 \times 10^3 \Omega^{-1} \text{cm}^{-1} = -\sigma_{int}$$

$$\sigma_{ah} = -(\alpha\sigma_{xx0}^{-1} + \beta\sigma_{xx0}^{-2})\sigma_{xx}^2 - b$$

at 5K

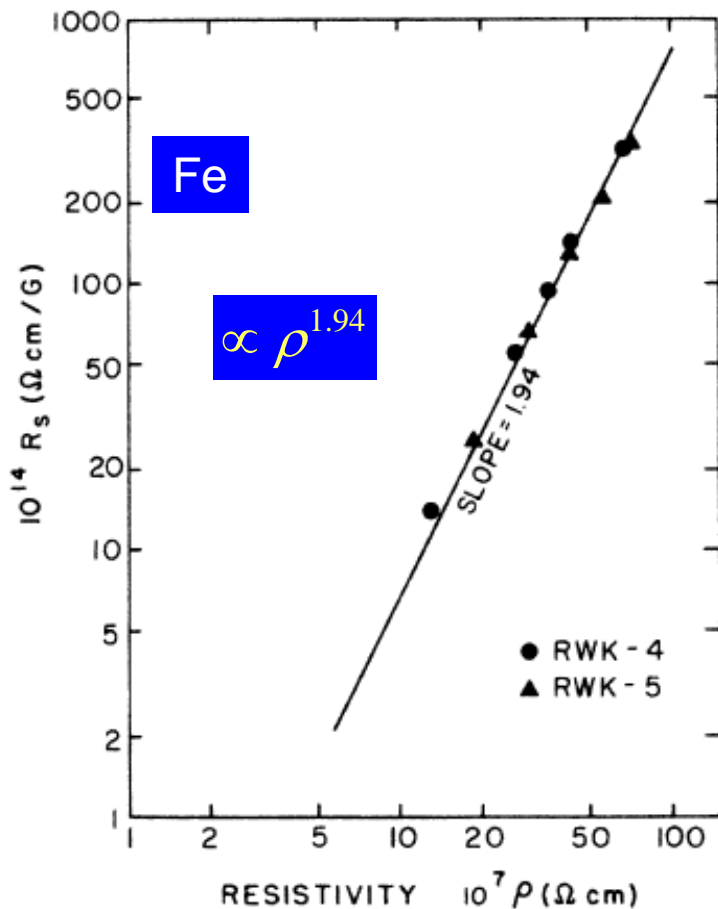
$$-\sigma_{ah0} = \alpha\sigma_{xx0} + (\beta + b)$$



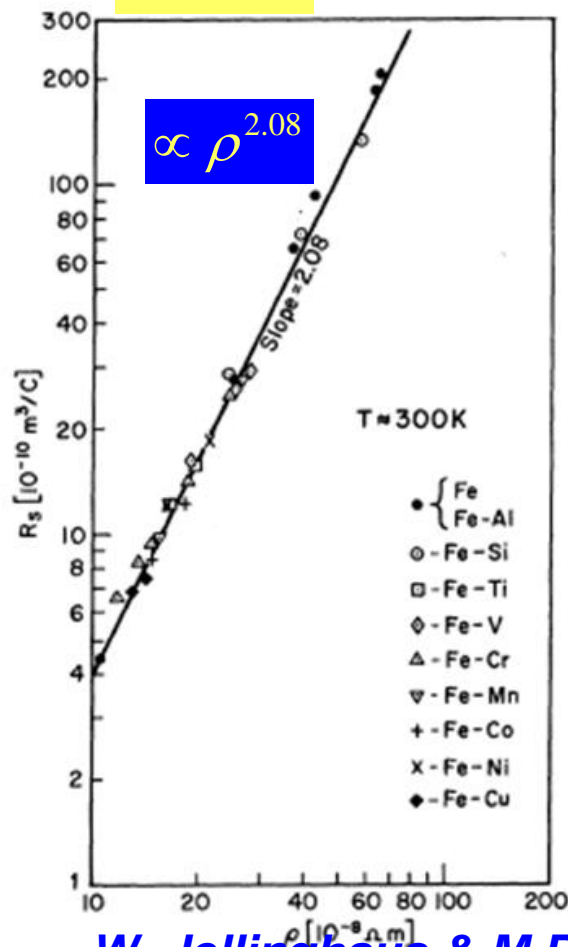
$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + b\rho_{xx}^2$$

Category 1:

$$b\rho_{xx}^2$$



R. W. Klaffy & R.V. Coleman,
Phy. Rev B. 10, 2915 (1974).

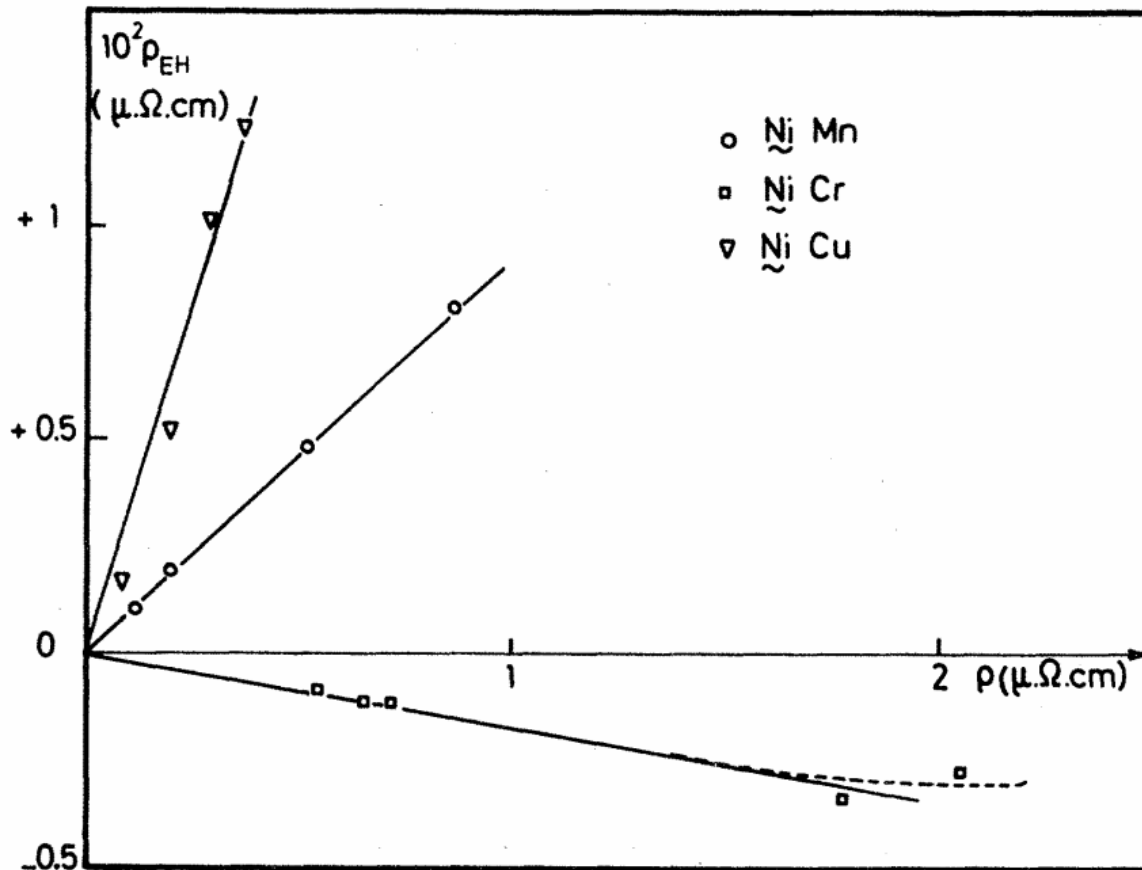


W. Jellinghaus & M.P. DeAndres,
Ann. Physik, 462, 189 (1961)

$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + b\rho_{xx}^2$$

Category 2:

$$a\rho_{xx}$$

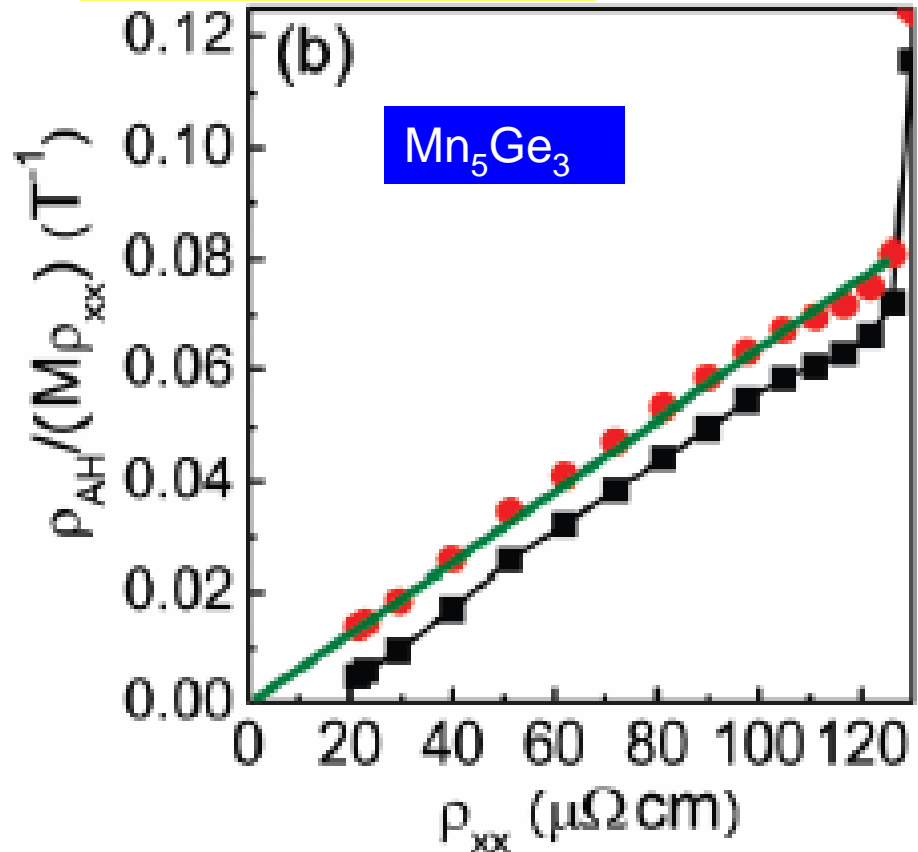
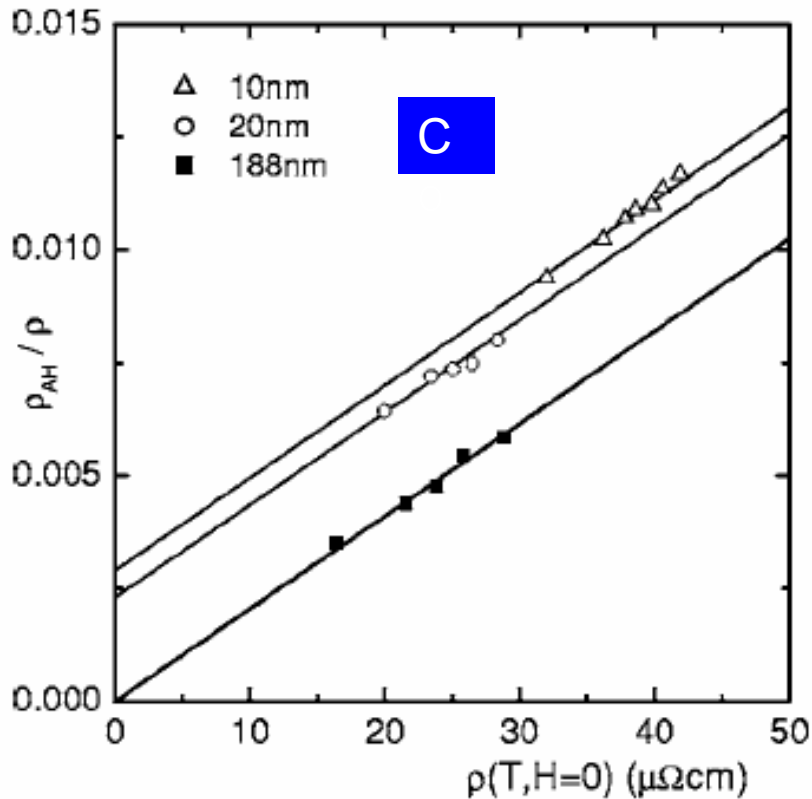


A. Fert and O. Jaoul, PRL. 28, 303 (1972)

$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + b\rho_{xx}^2$$

Category 3:

$$a\rho_{xx} + b\rho_{xx}^2$$



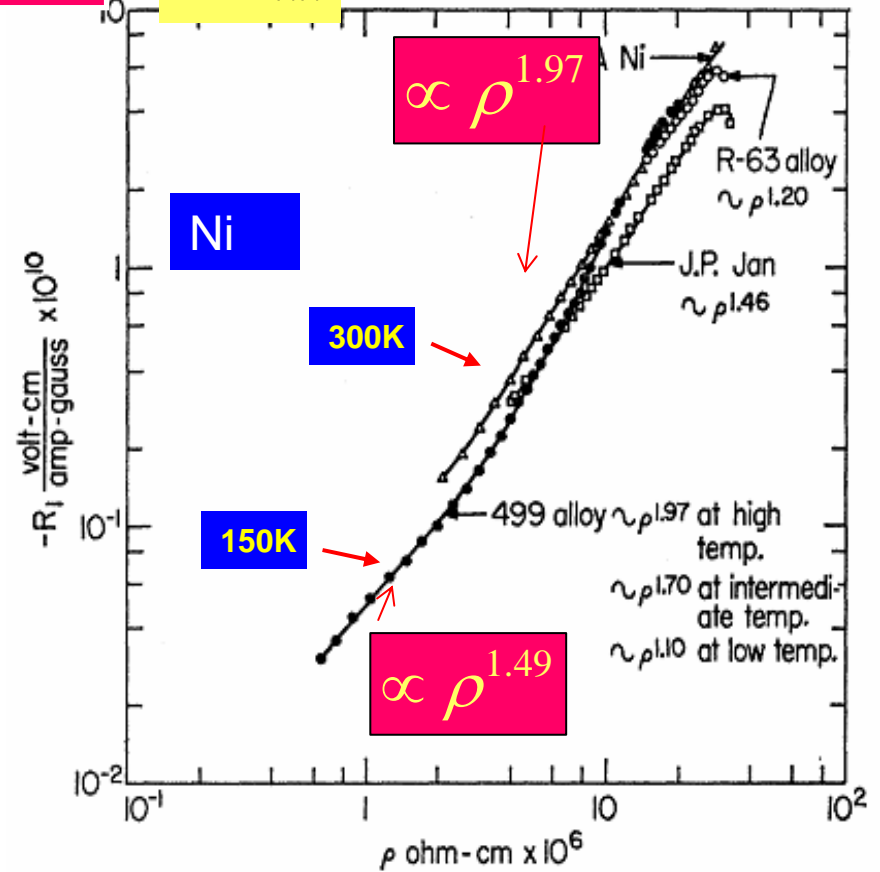
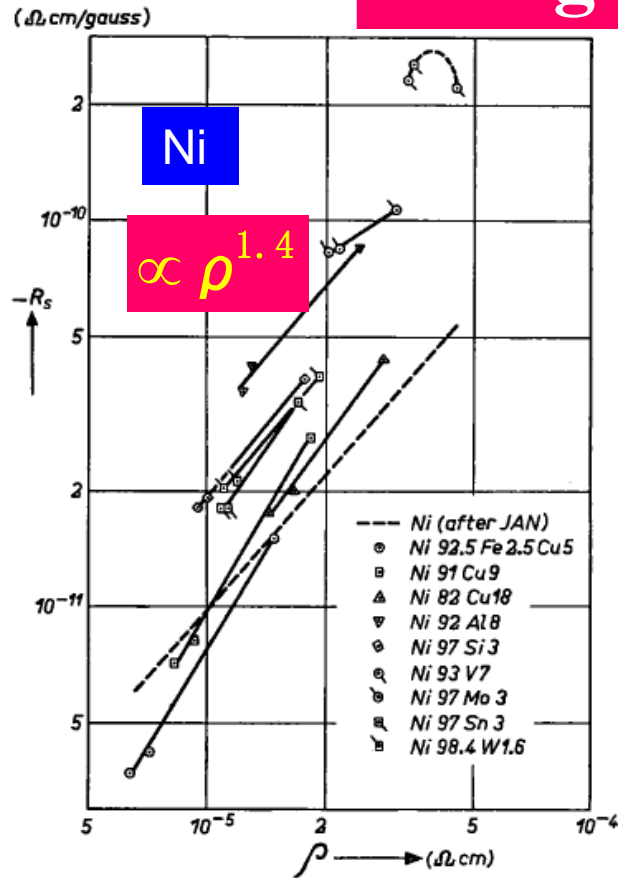
*J. Kotzler and W. Gil,
Phys. Rev. B 72, 060412 (2005)*

C. G. Zeng, et, al, PRL. 96, 037204 (2006)

$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + b\rho_{xx}^2$$

Category 4:

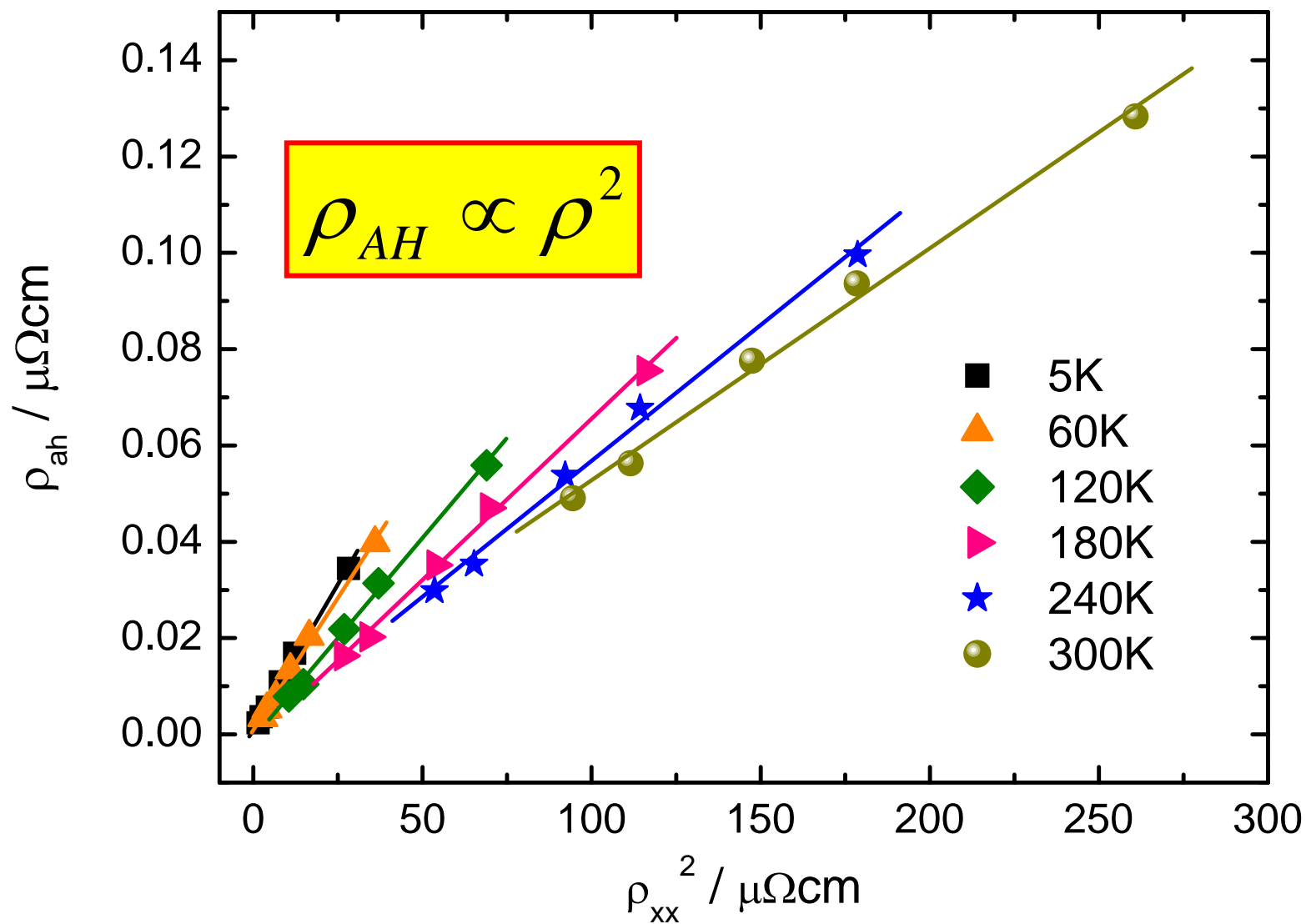
$$b\rho_{xx}^\alpha$$



J. Smit, *Physica* 24, 39 (1954)

J. Mavine, *Phys. Rev.* 123, 1273 (1961)

Anomalous Hall Resistivity vs Film Thickness



Conclusion 1:
Proper Scaling of the AHE ?

$$\rho_{ah} = f(\rho_{xx0}, \rho_{xx})$$

Conclusion 2:
Intrinsic and Extrinsic in the AHE ?

$$\rho_{ah} = \alpha\rho_{xx0} + \beta\rho_{xx0}^2 + b\rho_{xx}^2$$



THANK YOU