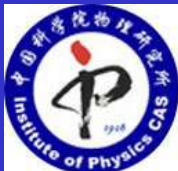


高温超导机理研究： 超越 *BCS* 理论框架

Hai-Hu Wen

National Lab for Superconductivity, Institute of Physics,
Chinese Academy of Sciences, Beijing, China

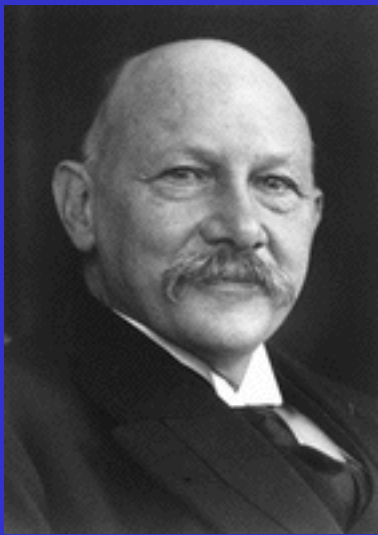
北京大学物理学院 2007



Collaborators:

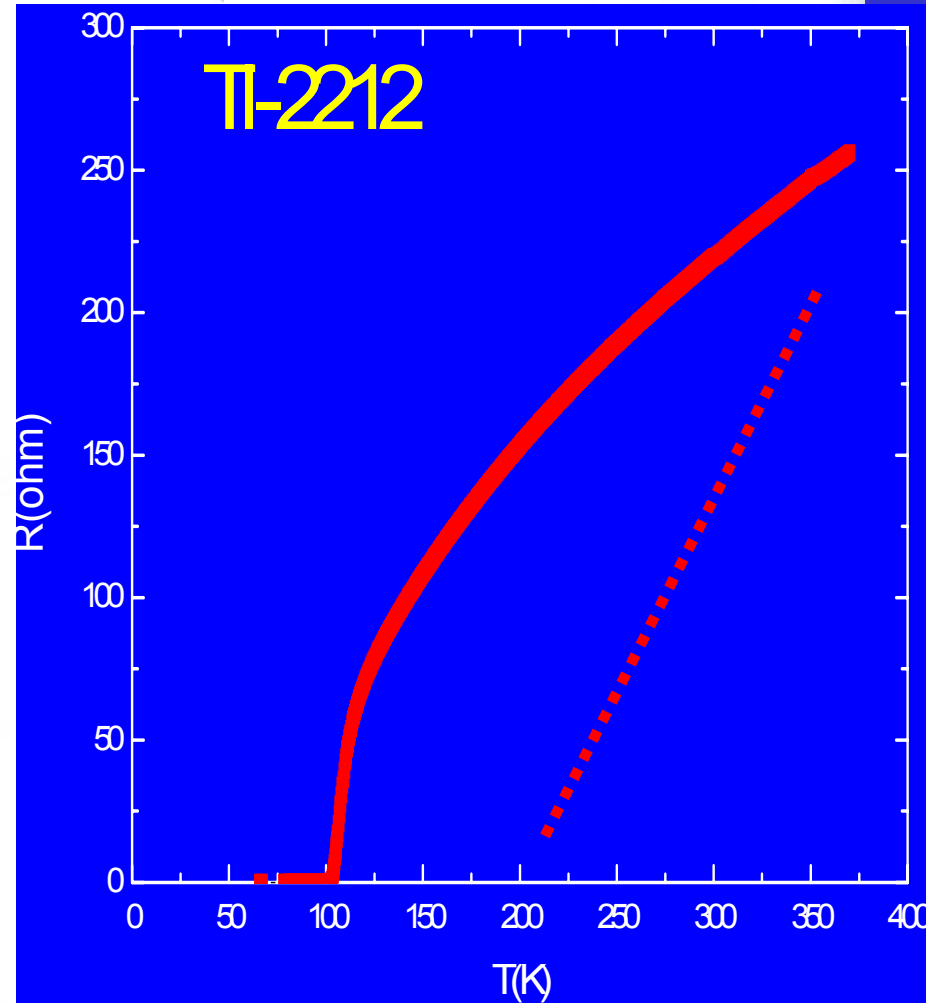
- IOP, CAS, China: Yue Wang, Lei Shan, Hong Gao, Zhiyong Liu, Fang Zhou
- MIT, USA: Xiao-Gang Wen
- Osaka Univ., Japan: Yoichi Ando's group (LSCO, Optimal doped)
- Tohoku Univ., Japan: Yoichi Tanabe, Tadashi Adachi, and Yoji Koike (LSCO, Overdoped)

零电阻特性

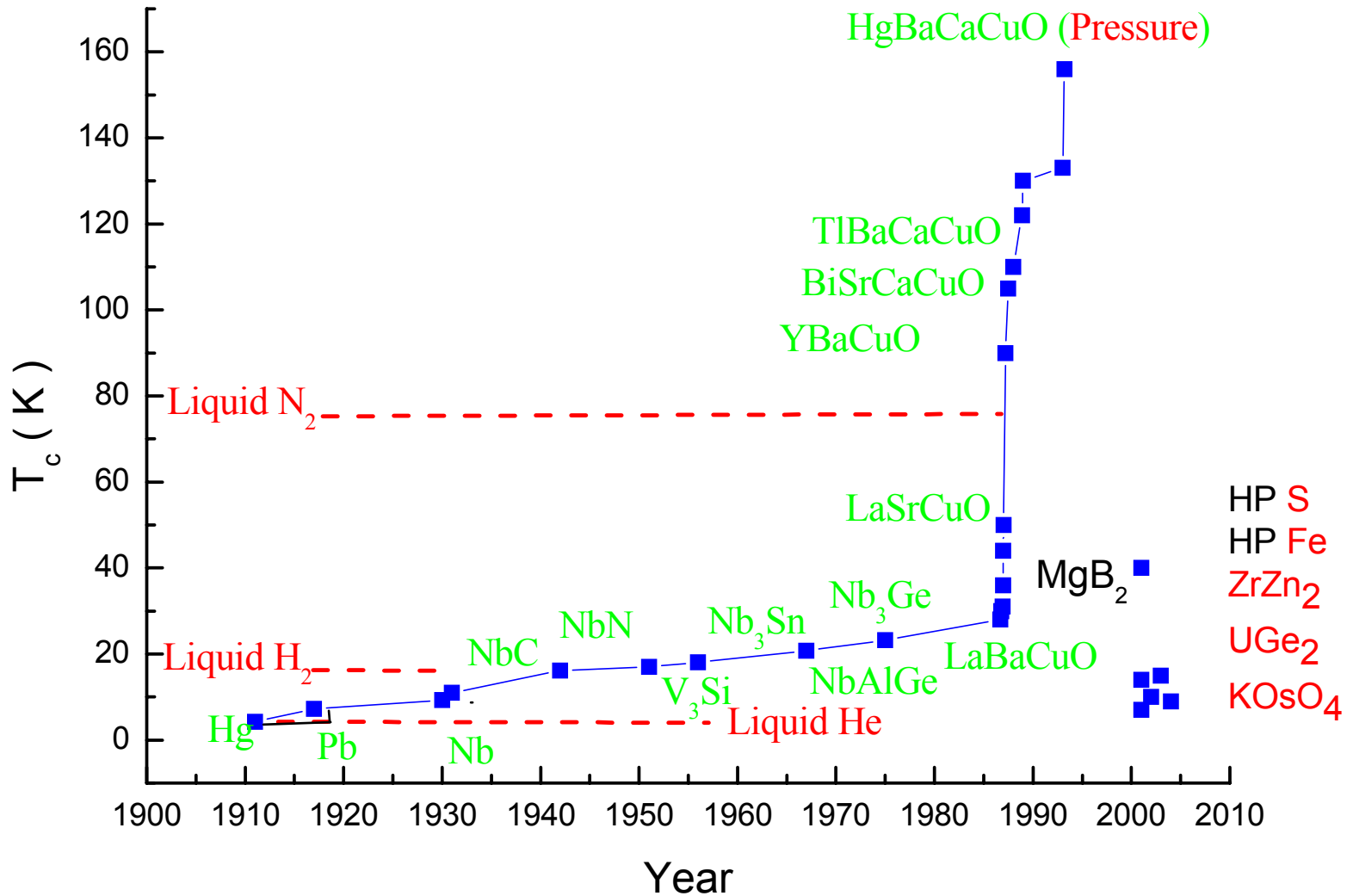


1913年获得诺贝尔物理学奖

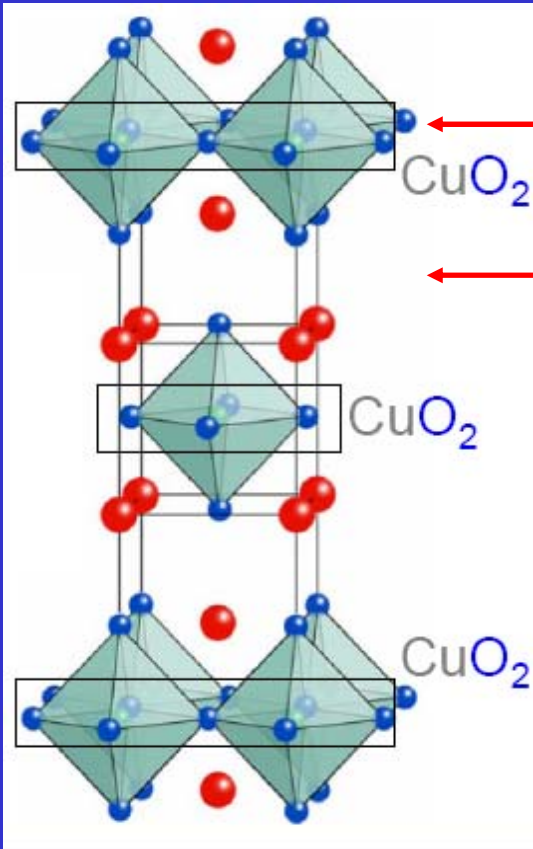
1908 年荷兰 Leiden 大学的 Kamerling Onnes 小组将 He 液化，1911年测量水银 Hg 的电阻温度曲线时发现在 4.2 K（零下269度）左右电阻突然消失。此性质被命名为超导性。随后发现了大批的单元素金属超导体。



超导转变温度与时间的关系

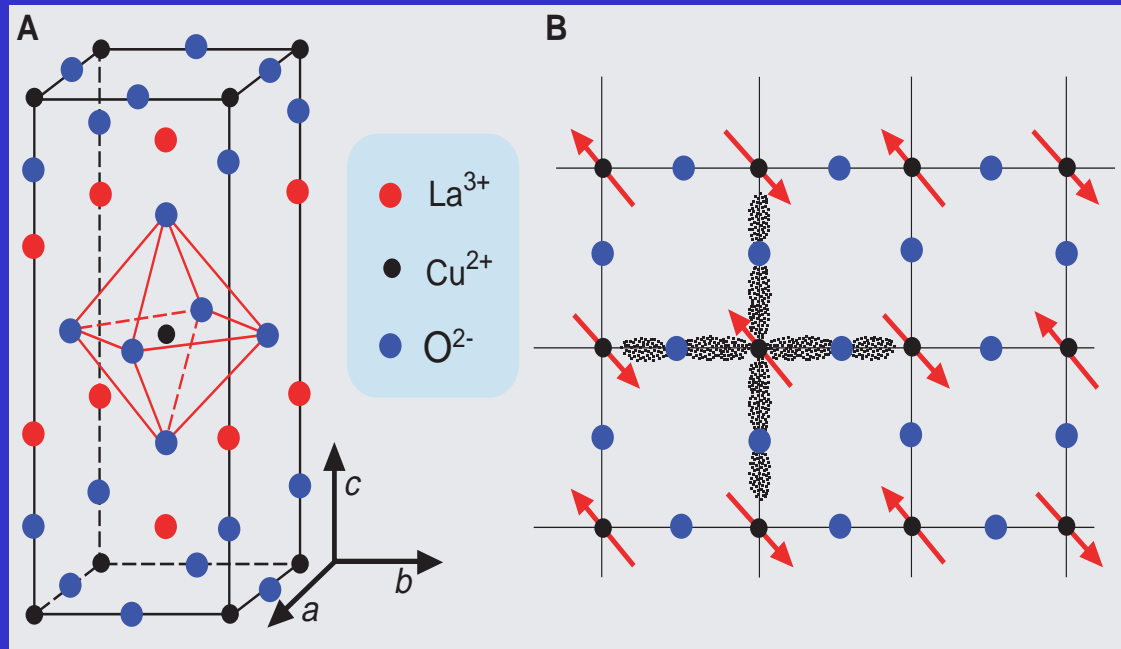


Structure of $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$: Doped Mott Insulator

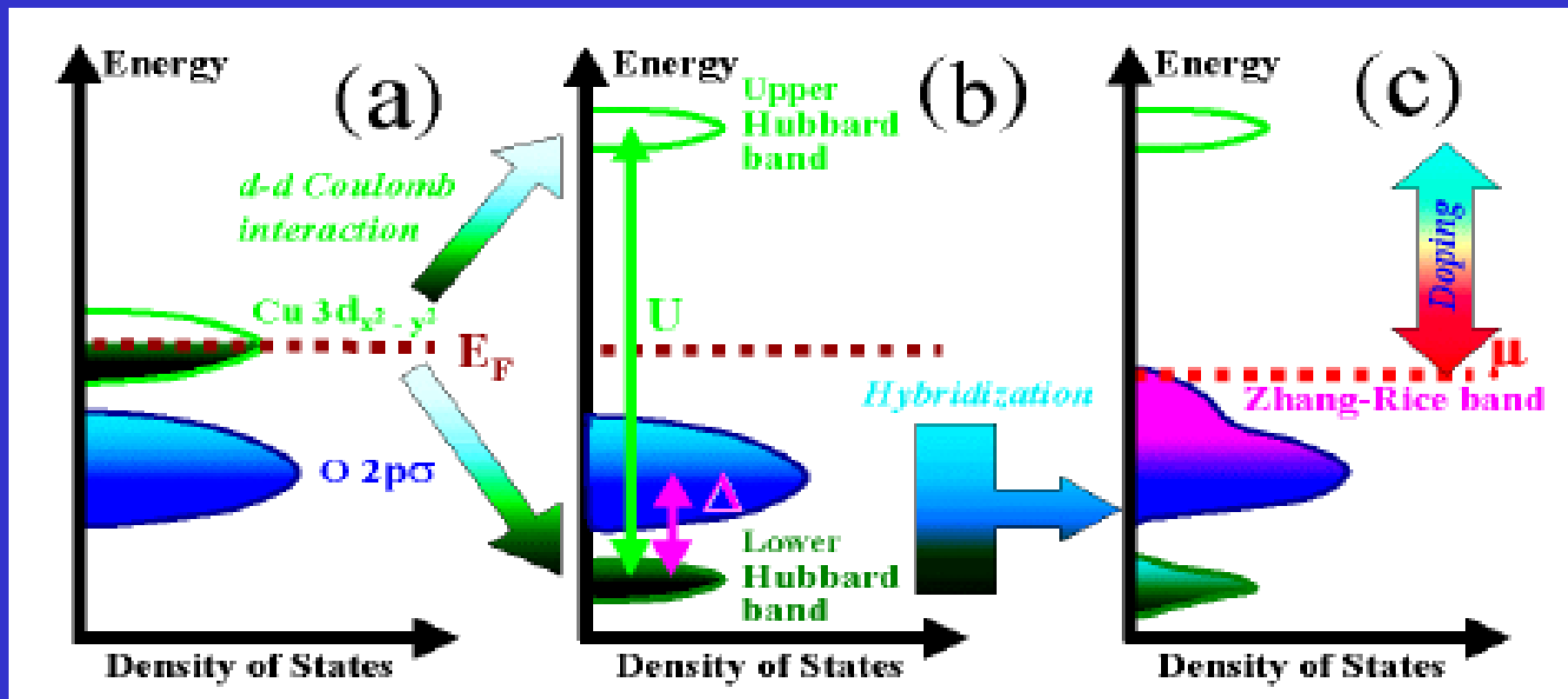


Cu-O plane

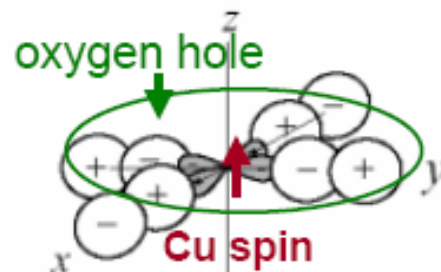
Charge Reservoir



空穴掺杂氧化物超导体电子能带图示



Zhang-Rice singlet





Temperature

D-wave
or S-wave ?

D-wave pairing

AF

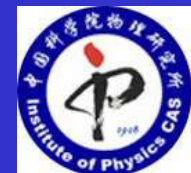
SC

SC

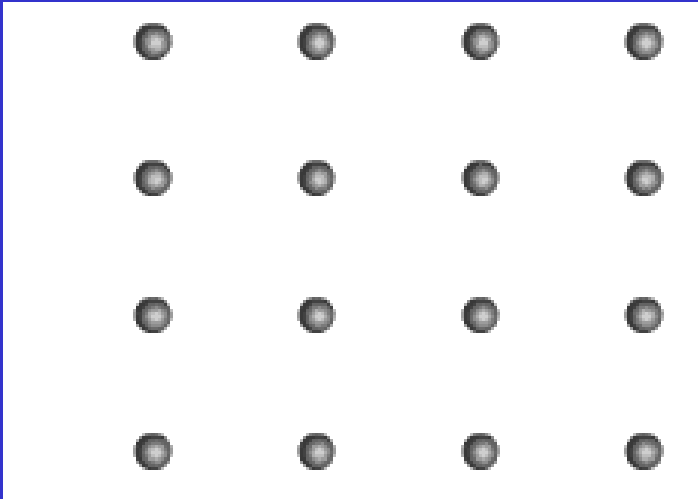
Electron

doping

Hole



BCS Theory : electron-ph interaction

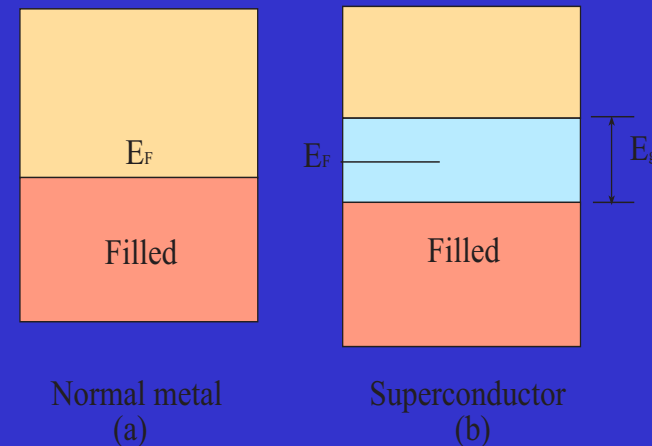


$$E \approx 2 E_F - 2 \hbar \omega_c e^{-2 / N(0) V}$$

$$W_{BCS}^0 - W_n^0 = -\frac{1}{2} N(E_F^0) \Delta^2$$

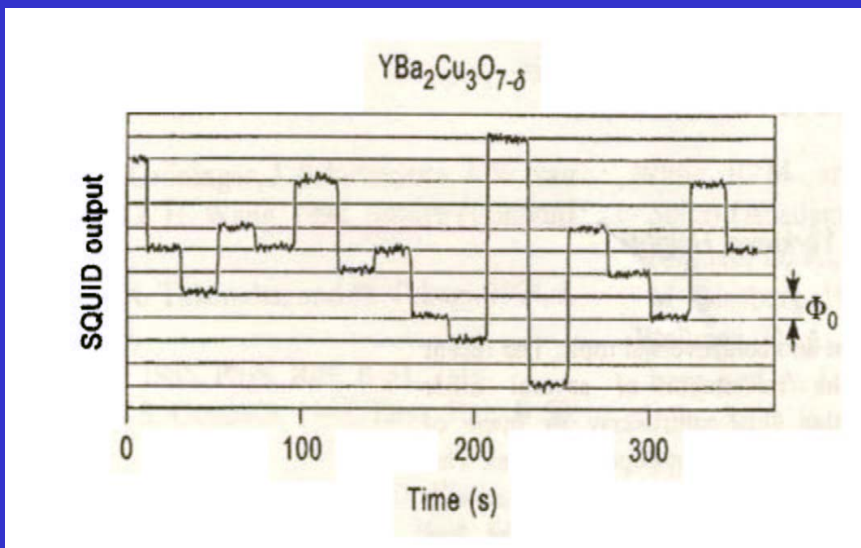
$$\Delta / k_B T_c = 1.76 \quad (\text{weak coupling s-wave})$$

$$\Delta / k_B T_c = 2.14 \quad (\text{weak coupling d-wave})$$



Symmetry of the Superconducting Order Parameter

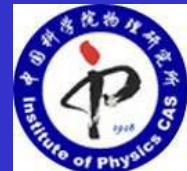
- High T_c superconductors: Cooper pairs: single flux with $2e$ as carriers in the superfluid condensate:



$$\Phi = h / 2e$$

C. E. Gough, et al.,
Nature 326, 855 (1987).

- Pauli exclusion principle: two electrons cannot occupy the same quantum state.



Spin and Orbital Angular Momentum of two Bounded Electrons

Spin State	S	Description	Orbital Number <i>l</i>
Spin Singlet	0	$\uparrow(1)\downarrow(2) - \uparrow(2)\downarrow(1)$	0 (s-wave) 2 (d-wave)
Spin Triplet	1	$\uparrow(1)\uparrow(2)$ $\downarrow(1)\downarrow(2)$ $\uparrow(1)\downarrow(2) + \uparrow(2)\downarrow(1)$	1 (p-wave)

Possible symmetries of curate high- T_c superconductors:

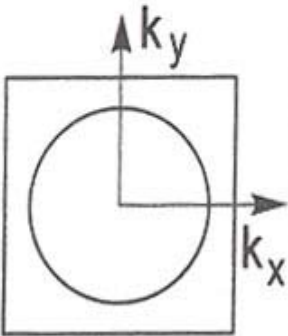
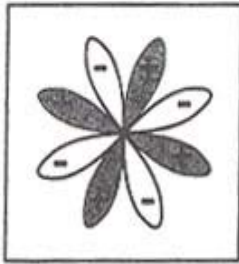
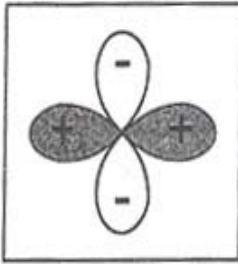
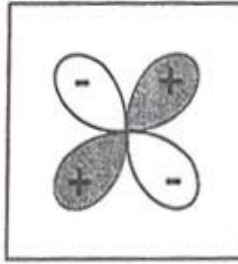
Group-theoretic notation	A_{1g}	A_{2g}	B_{1g}	B_{2g}
Order Parameter basis function	constant	$xy(x^2-y^2)$	x^2-y^2	xy
Wavefunction name	s-wave	g	$d_{x^2-y^2}$	d_{xy}
Schematic representation of $\Delta(k)$ in B.Z.				

Table 1

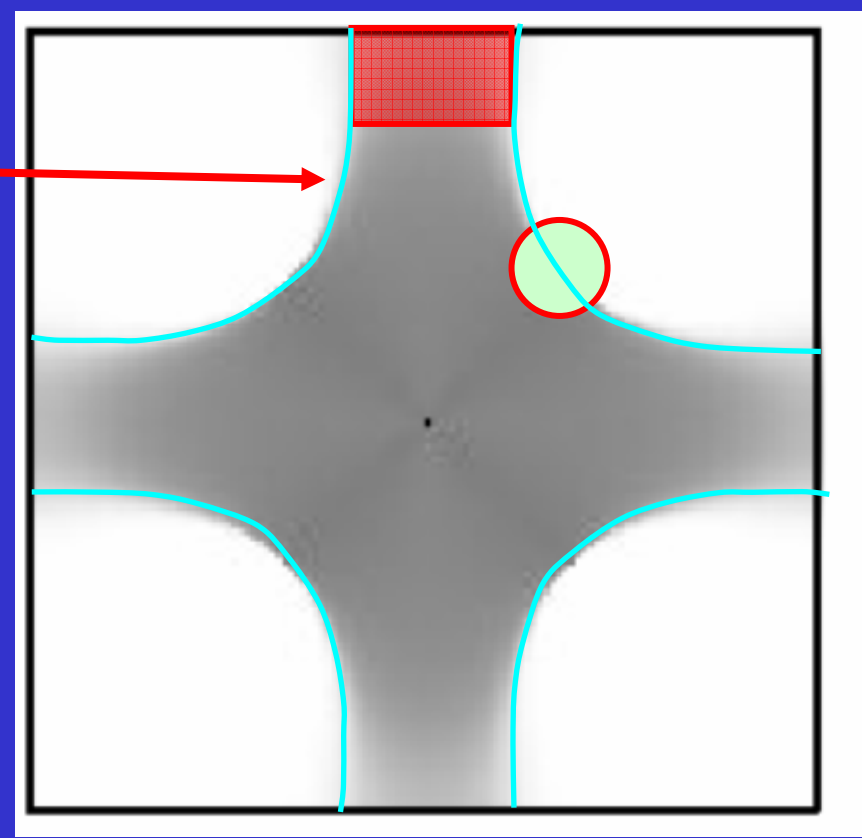
Possible even-parity singlet pair states in a square lattice (point group C_{4v})

From Q. H. Wang

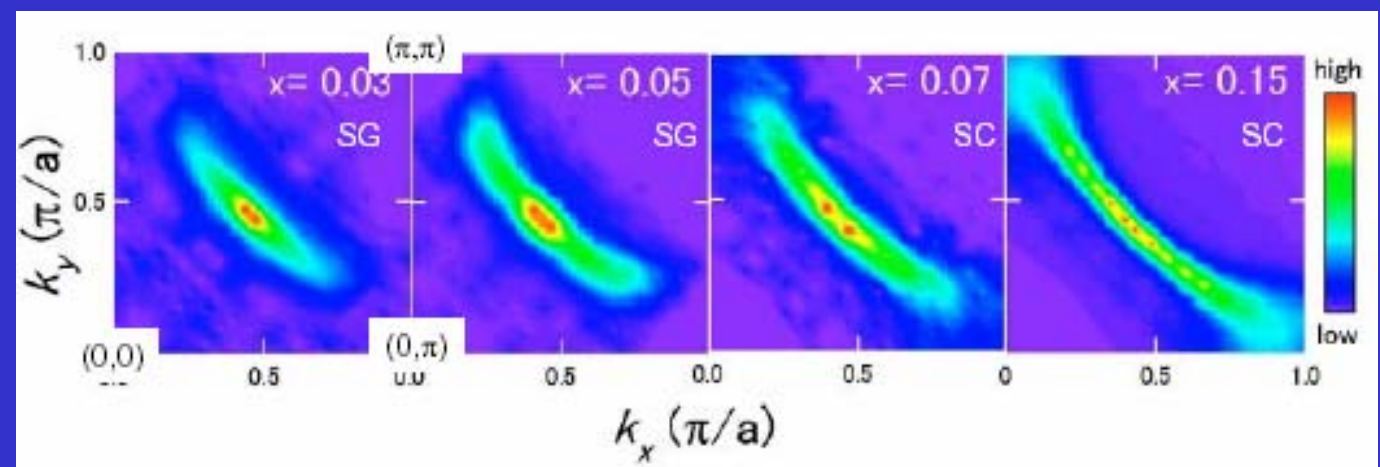
Anti-nodal region

Fermi surface

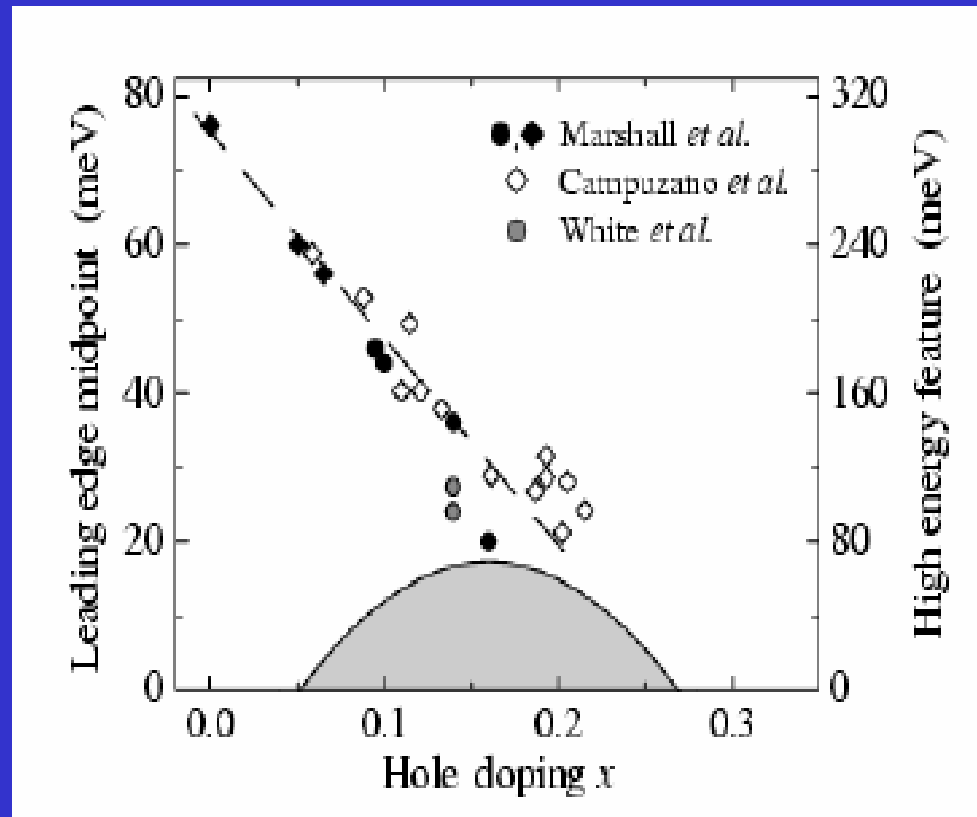
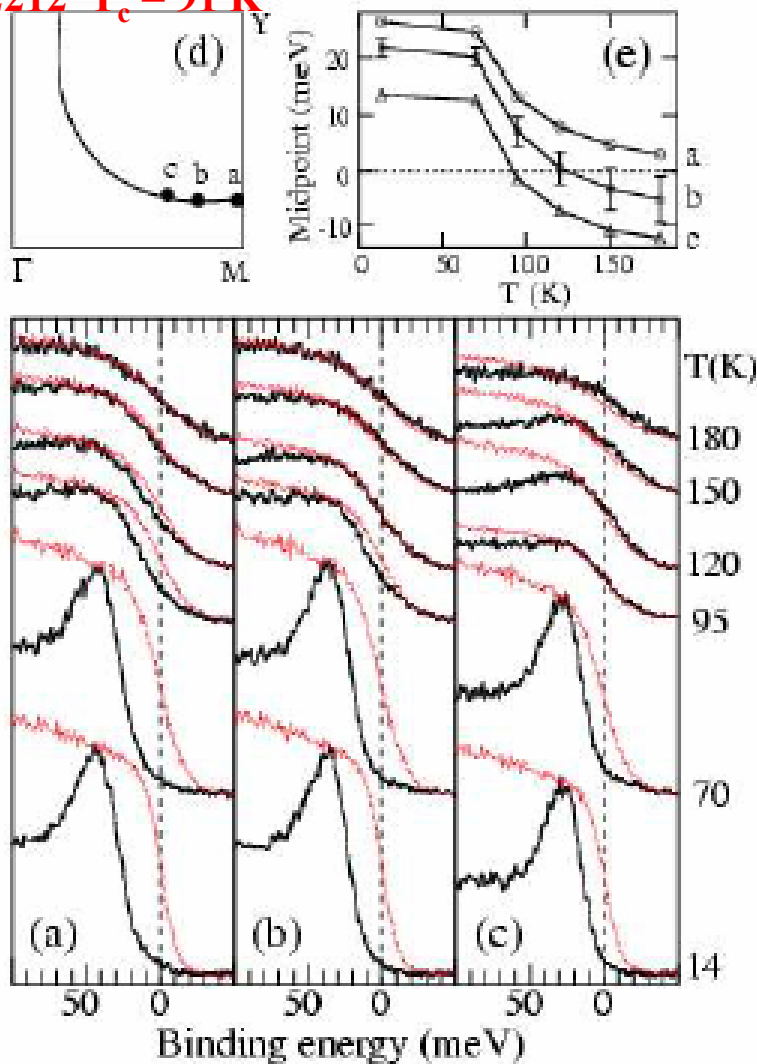
Nodal region



T. Yoshida et al.
LaSrCuO



Bi-2212 $T_c = 91$ K

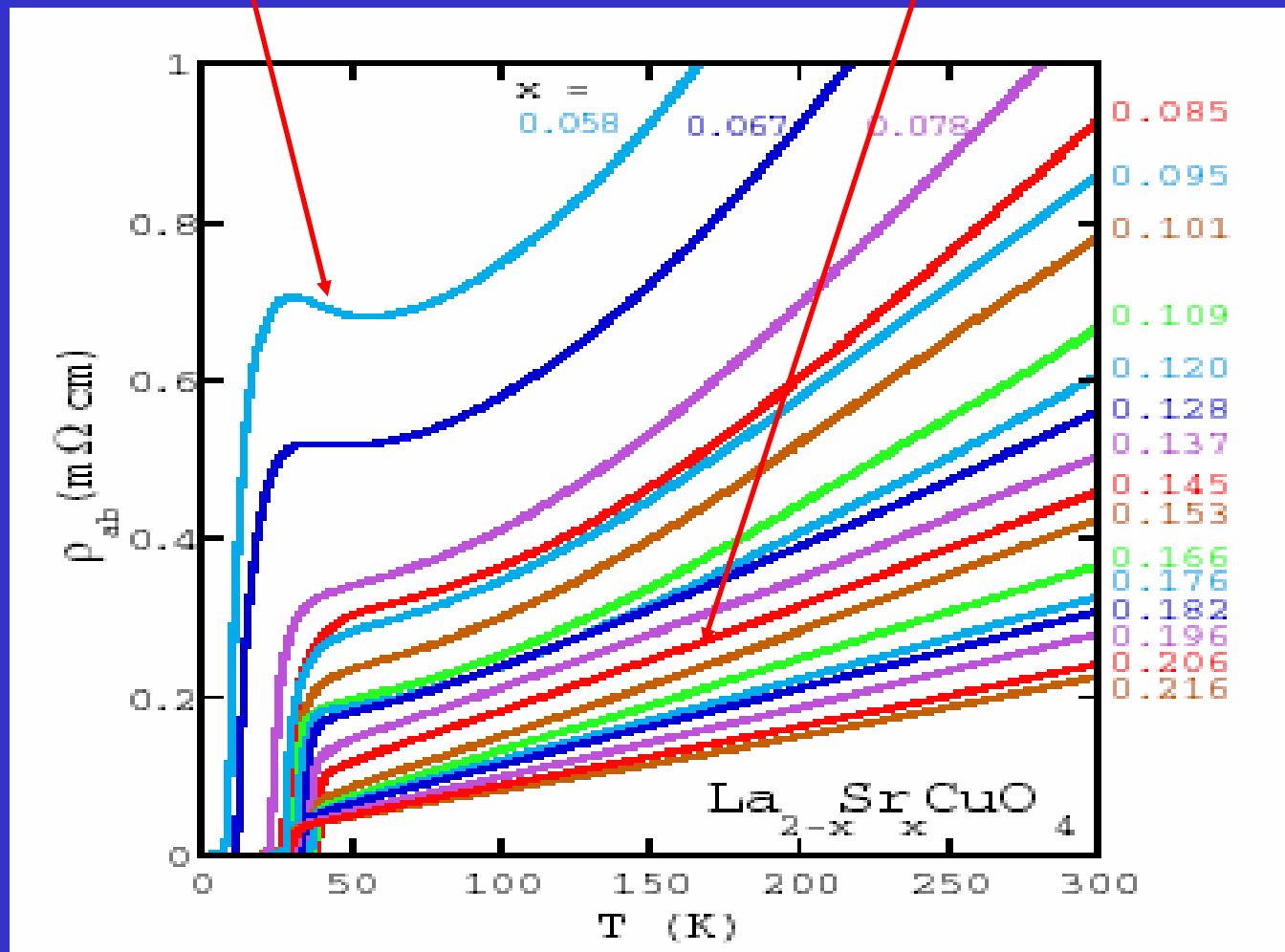


Andrea Damascelli, Zhi-Xun Shen, Zahid Hussain, Rev. Mod. Phys. 75, 473-541(2003).

The pseudogap seems to have the similar symmetry as the gap in the superconducting state, i.e., $d_{x^2-y^2}$.

欠掺杂区的电阻上翘

最佳掺杂的超线性



Komiya, ..., Ando, PRL94, 207004(2005)

非常规线性电阻行为

Novel Nernst effect in HTS

Z. A. Xu, N. P. Ong, Y. Wang, T. Kakeshita, S. Uchida, *Nature* 406, 486-488(2000).

Patrick A. Lee, *Nature* 406, 467(2000)

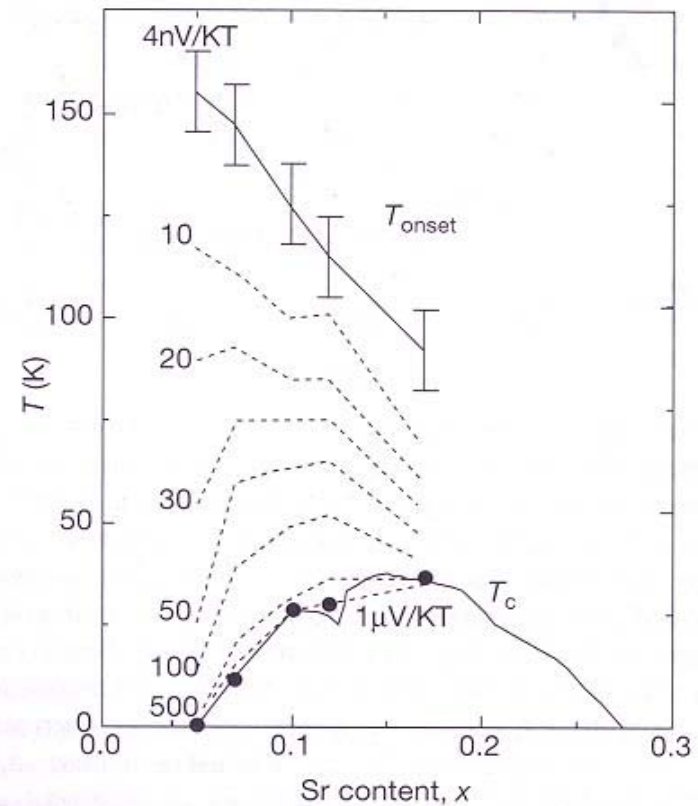
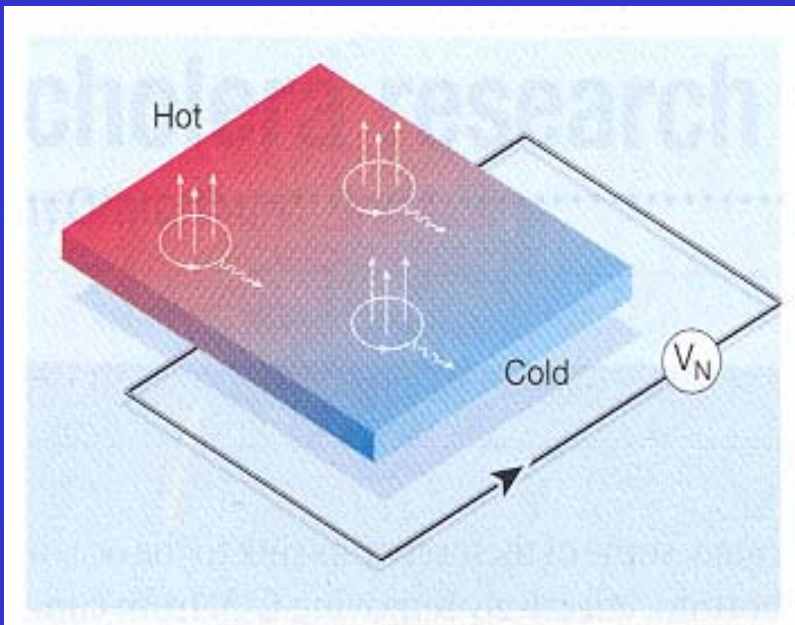
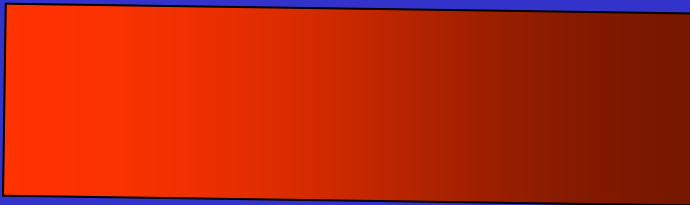


Figure 4 Contour plot of $(\nu - \nu_n)$ versus x in the phase diagram of LSCO. The contour plot displays how high in T the vortex-like excitations extend for each value of x . The upper solid line T_{onset} is the contour set by our resolution. The pseudogap T^* estimated from heat capacity¹⁵ is about a factor of two larger than T_{onset} . Values of T_c in our samples (circles) match the T_c line (lower solid line) from Takagi *et al.*¹⁴ We note that the T_c line is roughly similar to the contour line $\nu = 1 \mu\text{V}/\text{KT}$.

Nernst signal due to normal electrons is almost zero

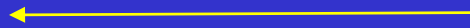
Thermal flow



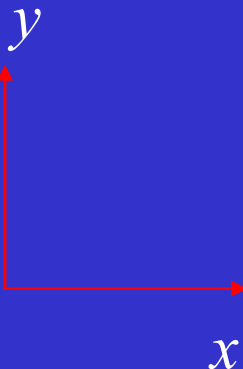
$$H = 0$$

$$j_x = \sigma E_x + \alpha (-\nabla_x T)$$

$$E_x = -(\alpha / \sigma)(-\nabla_x T)$$



Anti-electric-field



$$v_N = \frac{E_y}{|\nabla_x T| B} = \left[\frac{\alpha_{xy}}{\sigma} - S \frac{\sigma_{xy}}{\sigma} \right] / B$$
$$= \left[\frac{\alpha_{xy}}{\sigma} - S \tan \theta \right] / B = 0$$

$H \neq 0$, No electrons flow,
thus NO transverse voltage.

Nernst effect due to *flux motion*

$$F_{th} = -S_{\phi} \nabla_x T \longrightarrow \text{Thermal force}$$

$$F_{\eta} = -\eta v_{\phi} \longrightarrow \text{Dissipation force}$$

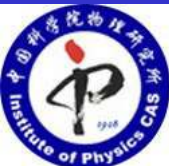
$$E_y = v_{\phi} B_z$$

Nernst voltage due to thermal drifting flux flow

$$E_y = \frac{\nabla_x TS_{\phi}}{\phi_0} \rho_f = \frac{\nabla_x TS_{\phi}}{\phi_0} \rho_n \frac{H}{H_{c2}(T)}$$

$$S_{\phi} = \left(\frac{\phi_0}{4\pi T} \right) (H_{c2}(T) - H) \frac{L(T)}{1.16(2\kappa^2 - 1)}$$

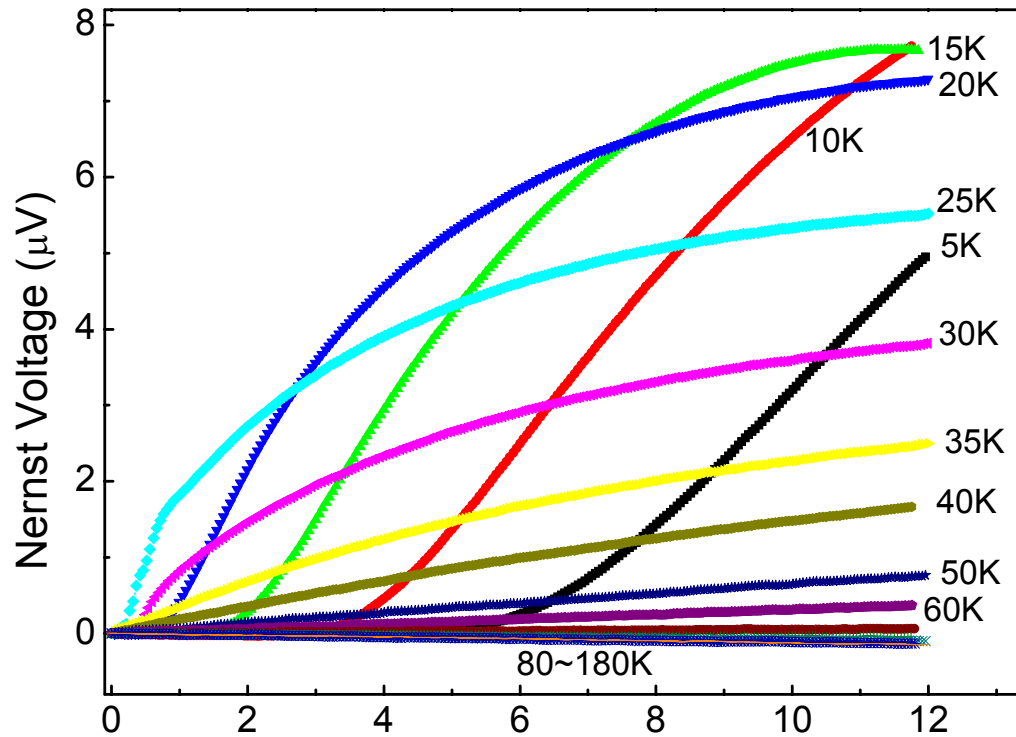
*K. Maki, Physica 55, 124
(1971).*



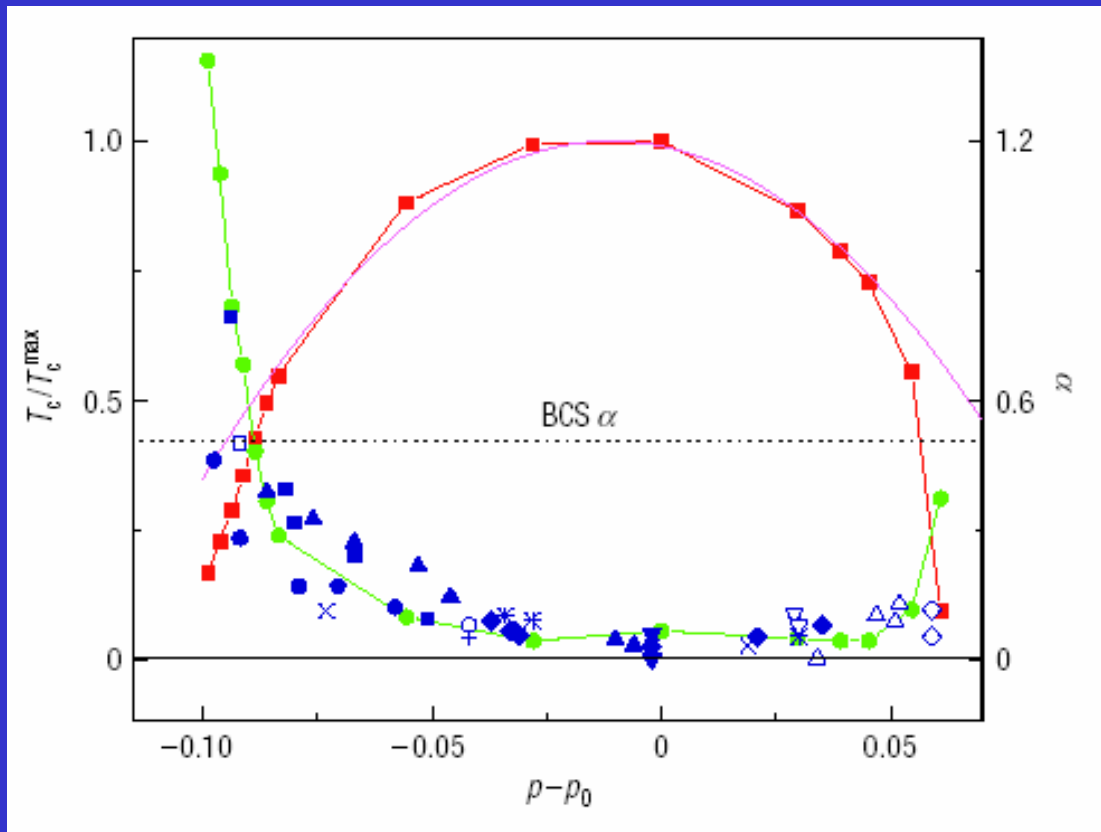
Thermal driven flux motion



Nernst Voltage



- *Z. Wang, H. H. Wen, PRB72, 054509(2005).*
- *H. H. Wen et al., EPL63, 583(2003)*



BCS 电声
子:

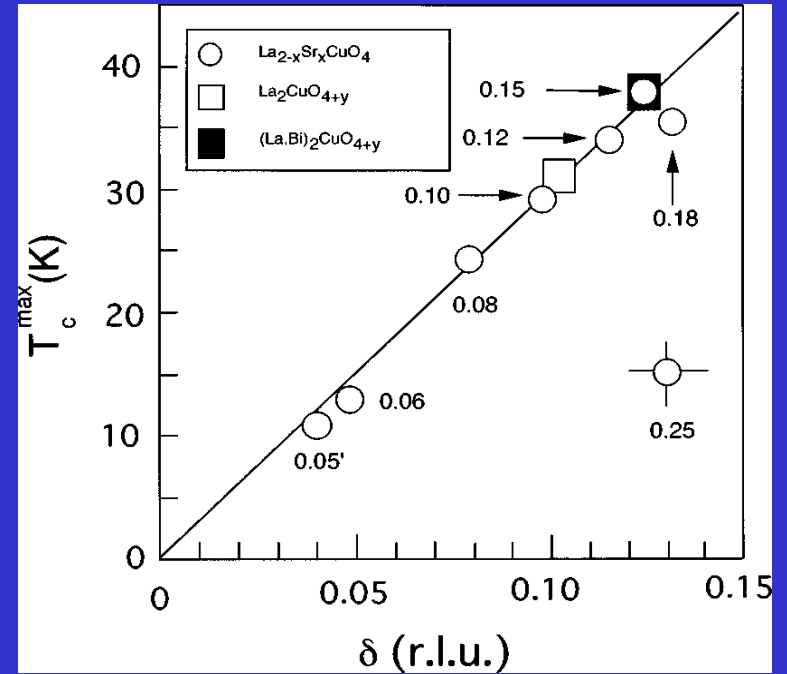
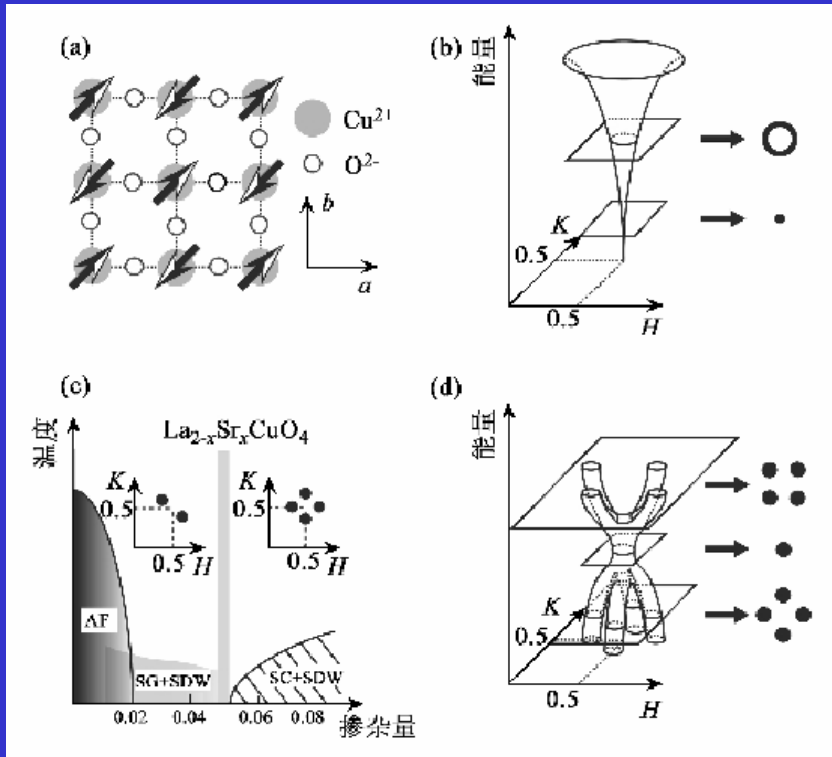
$$T_c M^\alpha = \text{常数}$$

$$\alpha = 0.5$$

反常的同位素效应:

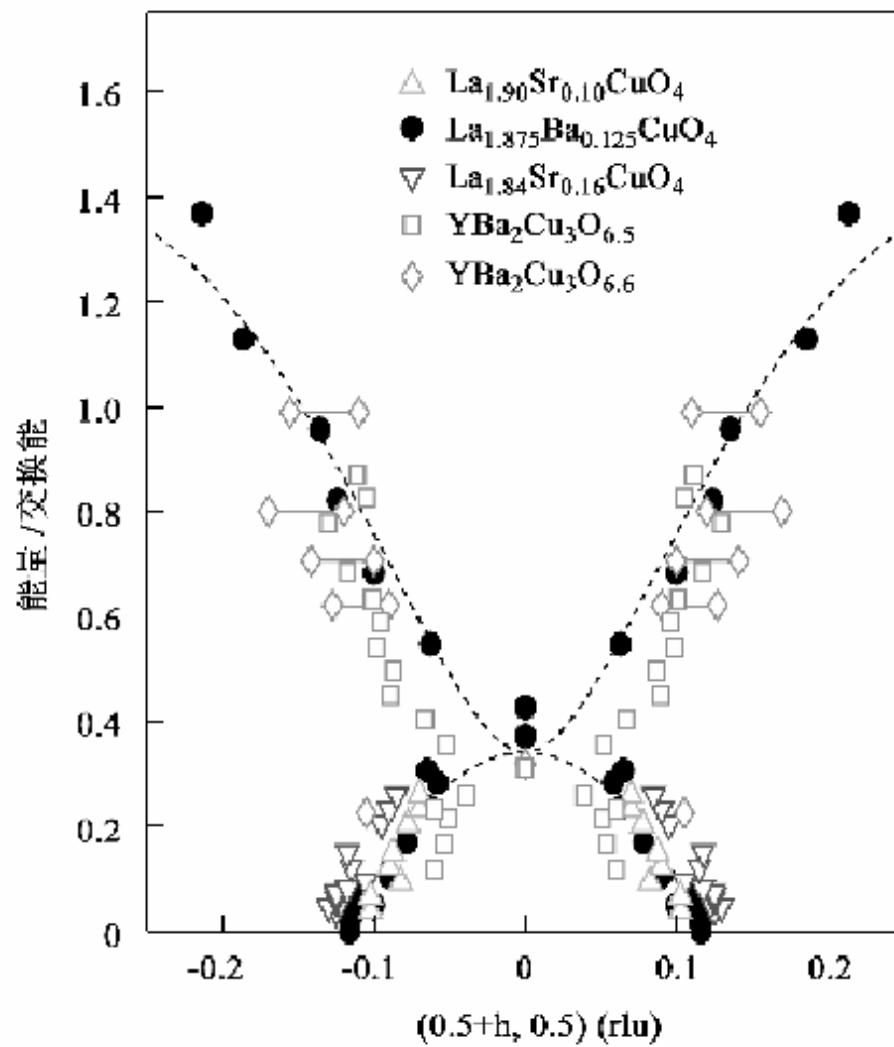
图取自: D. M. Newns and C. C. Tsuei, Naturephys.
3, 184 (2007).

高温超导体中特殊的自旋涨落谱

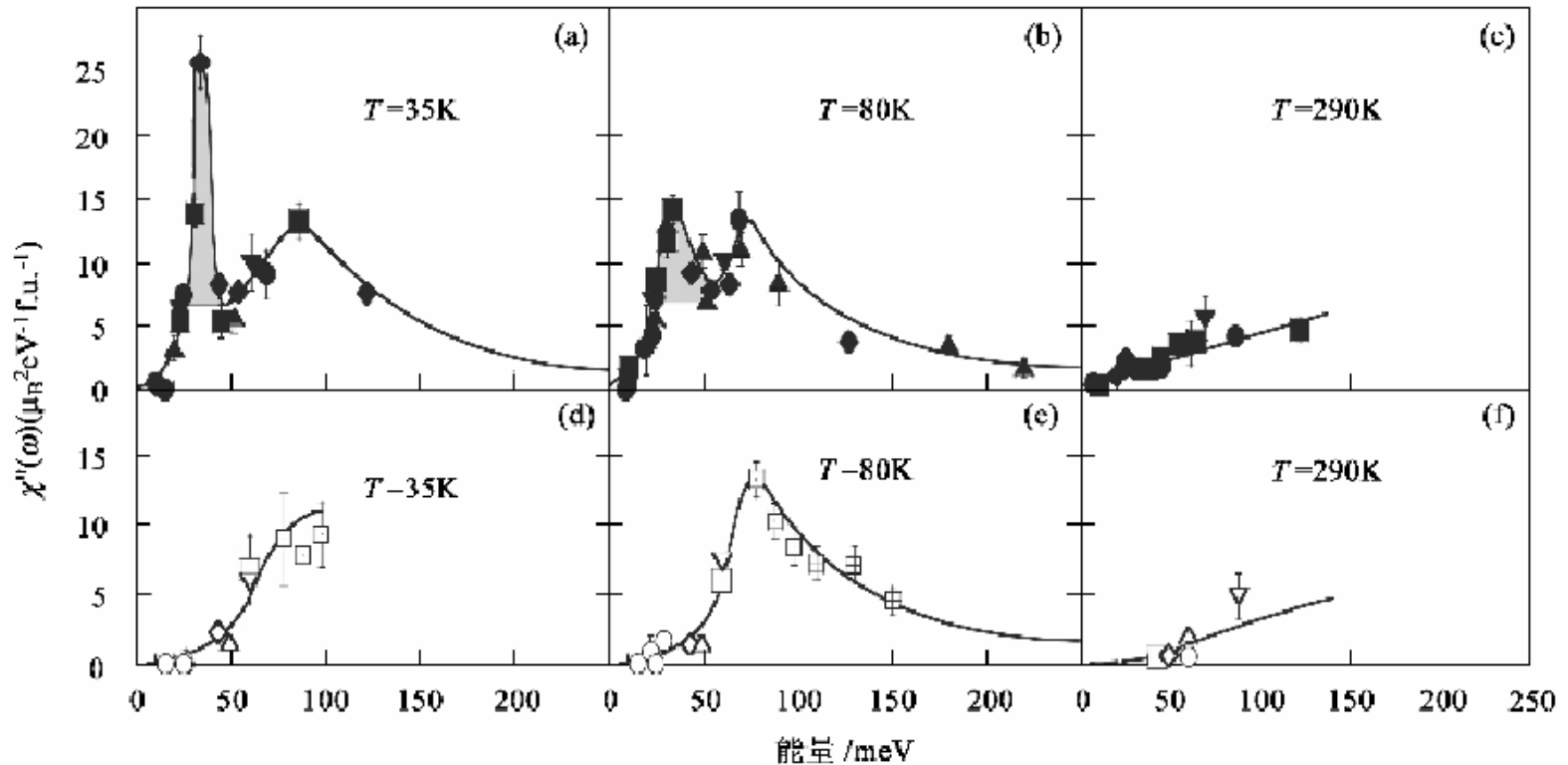


李世亮和戴鹏程，物理，
2006

**K. Yamada et al. PRB 57,
6165(1998).**



J. Tranquada, Nature 2006



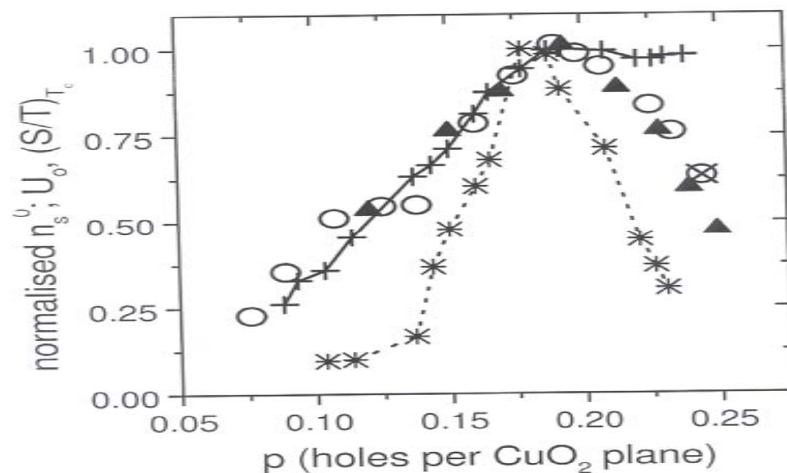
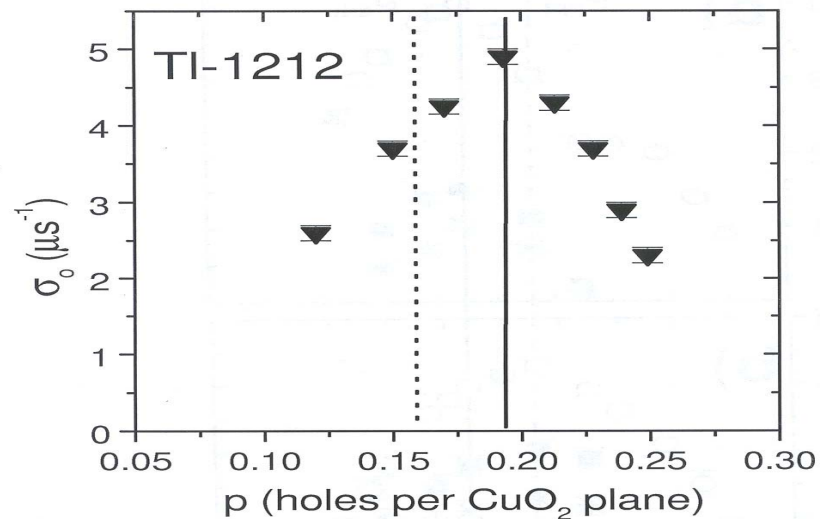
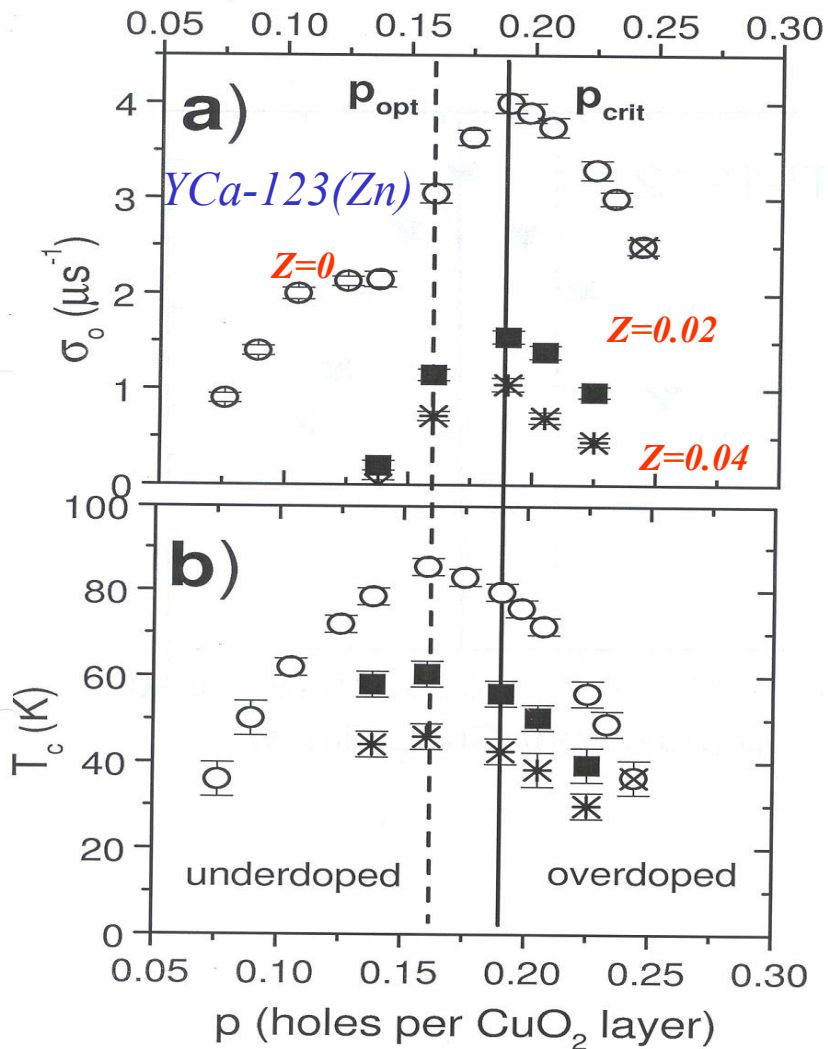
P. C. Dai et al, Science 284, 1344 (1999).

About the resonance peak there are two pictures:

1. Media (magnetic fluctuation) for electron pairing;
2. The response of the spin system when the d-wave superconducting gaps are formed.

Superfluid density and doping

C. Bernhard et al., PRL86, 1614(2001).



Electronic phase diagram of HTS

---BE —BCS Condensation Scheme

- Y. J. Uemura, *PRL*66, 2665(1991).
Nature 364, 605(1993). *Physica C* 282-287, 194(1997).

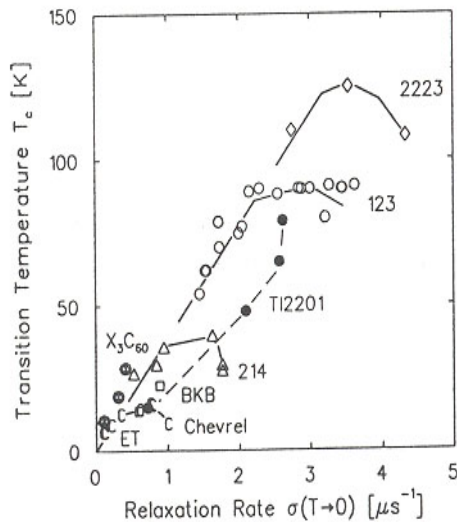


Figure 1. Muon spin relaxation rate $\sigma(T \rightarrow 0)$ in various superconductors plotted versus T_c [1-3].

Overdoped
Regime:
Swiss Cheese
Model

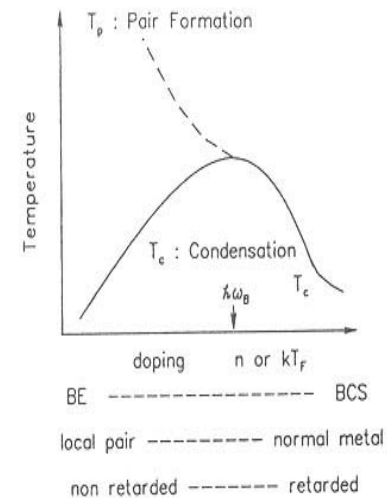
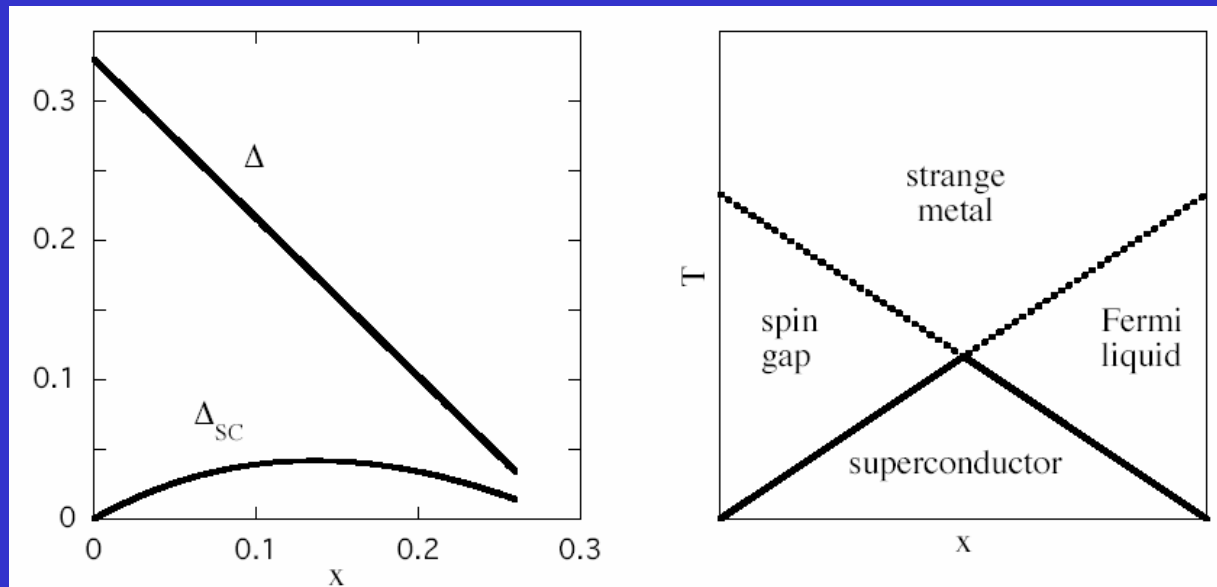
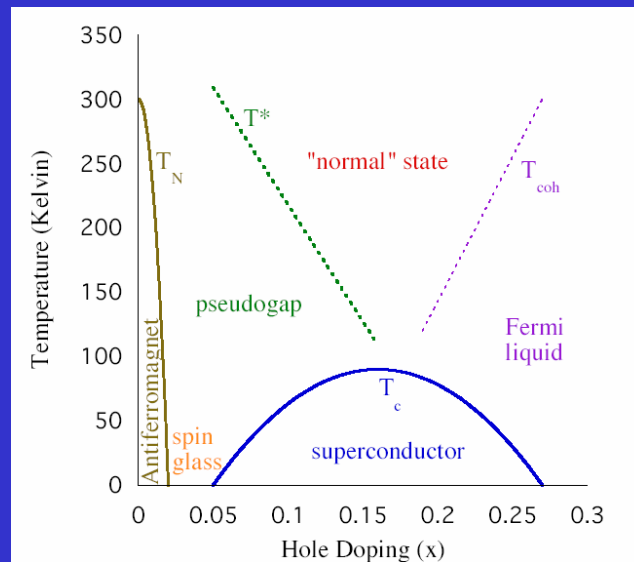


Figure 3. Phase diagram describing BE-BCS crossover with increasing carrier concentration n . This phase diagram can be mapped to that of the cuprates by assuming that the pseudo gap temperature T^* corresponds to the formation of normal-state pairs. [5,6]



V. J. Emery and S. A. Kivelson, Nature 374, 434 (1995).

围绕高温超导机理产生很多问题：

- 为什么有抛物线形状的 T_c vs. p 曲线？
- 能隙的起源是什么？其基态是什么？金属、非金属？能带论和费米面仍然适用吗？
- 能隙与超导的关系？敌人、竞争者、共存者或朋友？
- 为什么欠掺杂的能隙很大，但是超导温度很低？为什么在远高于超导转变温度之上仍然有强的能斯特信号？
- 最佳掺杂的正常态电阻为什么到非常高温是线性行为？
- 是电声子耦合还是其他耦合，或者不需要配对媒介？
- 磁涨落谱上面的共振峰对应什么物理？是自旋涨落配对吗？
- 高温超导离传统的 BCS 有多远？

Specific heat: A powerful tool for low energy excitation

$$C = \frac{dU}{dT} = C_e + C_{ph} + C_{spin}$$

$$\Delta T = \Delta T_c \left[1 - \exp\left(-\frac{t}{\tau}\right) \right]$$

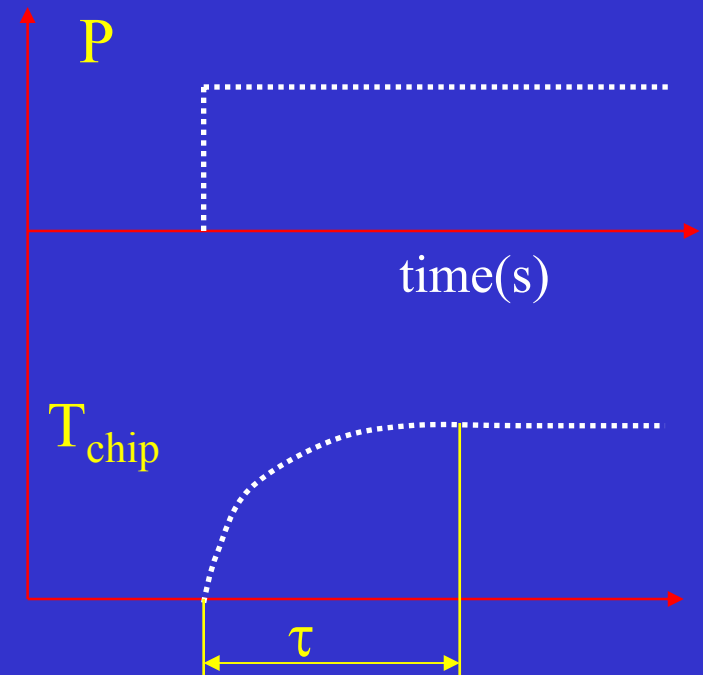
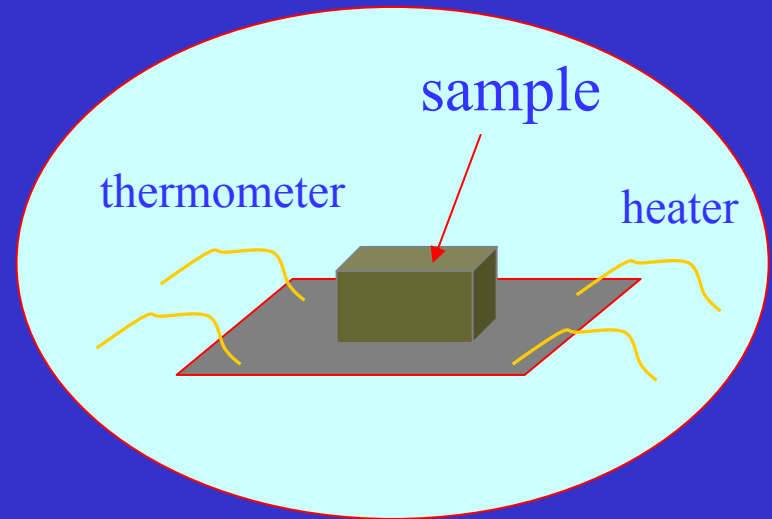
$$\tau = \frac{(C + C_{add})}{\kappa_w}$$

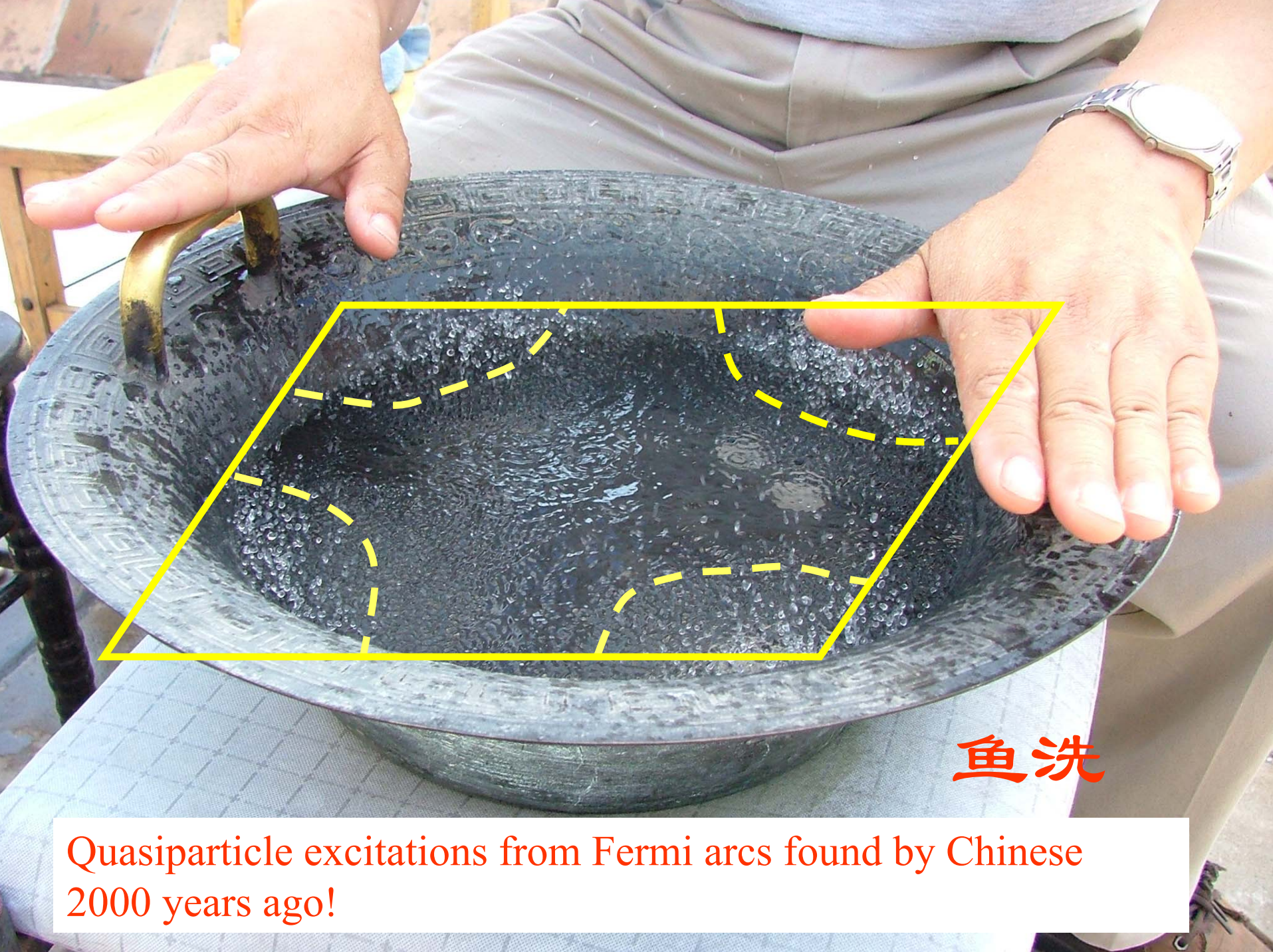
Sample: 1 to 50 mg.

T: 1.8 K to 400 K

H: 0 to 12 Tesla

C: 10 nJ / K





鱼洗

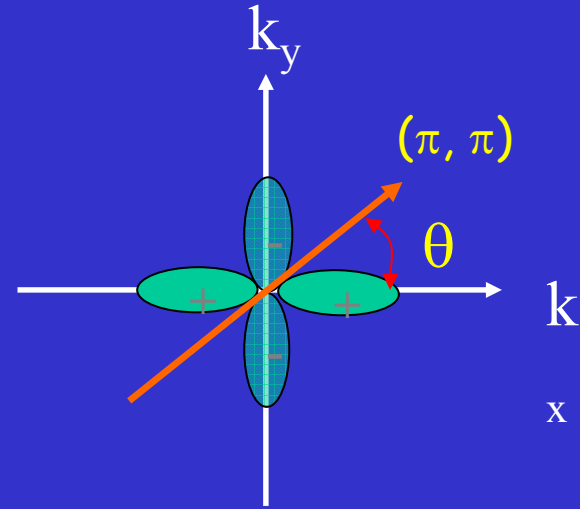
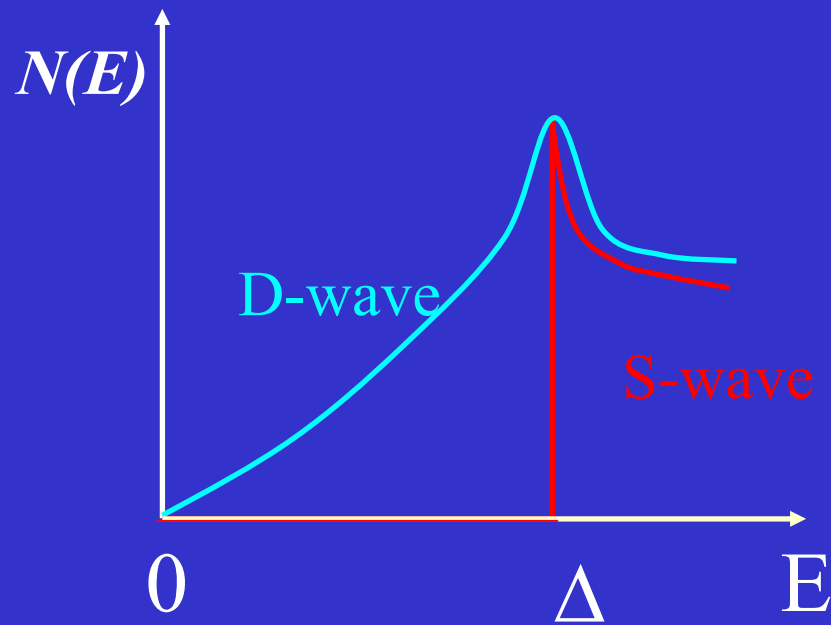
Quasiparticle excitations from Fermi arcs found by Chinese
2000 years ago!

S-wave

D-wave

$$N(E) = N(0) \operatorname{Re} \frac{E}{\sqrt{E^2 - \Delta^2}}$$

$$N(E) = N(0) \int \frac{d\theta}{2\pi} \operatorname{Re} \frac{E}{\sqrt{E^2 - \Delta_0^2 \cos^2(2\theta)}}$$

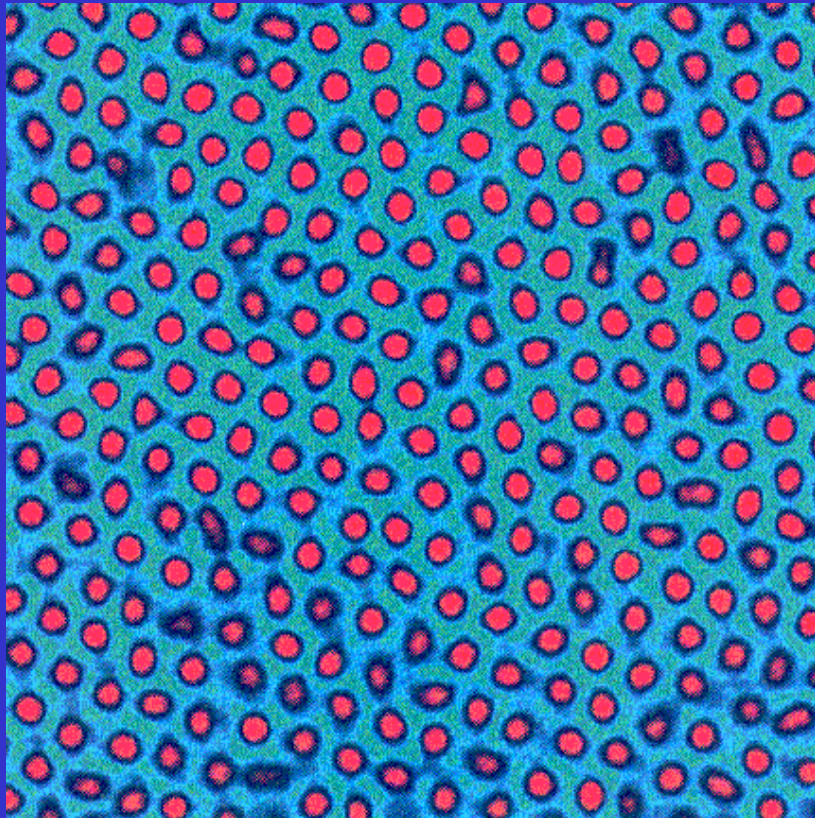


$$N(E) \propto E$$

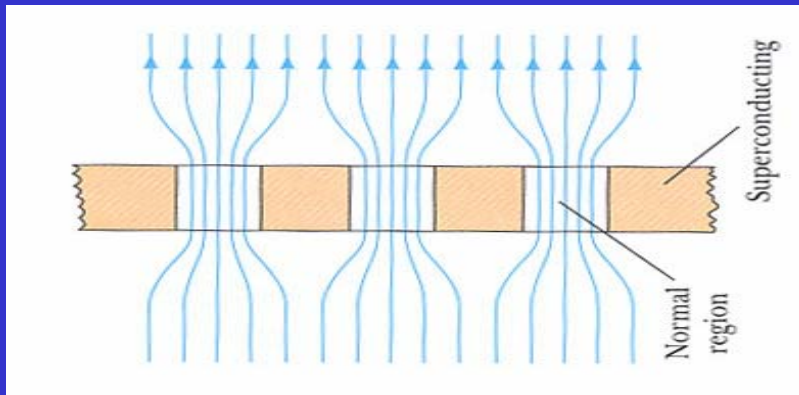
$$C_{\text{pure-s}} = a \gamma_n T_c \exp\left(-\frac{\Delta}{T}\right)$$

$$C_{\text{d-wave}} = a \frac{\gamma_n}{T_c} T^2$$

DOS in Mixed State with Vortices



1. **S-wave:** superconductor, vortex cores play as potential wells for low energy quasi-particles. **One component:** localized core state.
2. **D-wave:** two components for quasiparticles: Localized core state and de-localized extended states due to the nodes.



Symmetries → Quasiparticle excitation in mixed state

1. S-wave superconductor:

$$N(T, H) \approx a \gamma_n \frac{H}{H_{c2}}$$

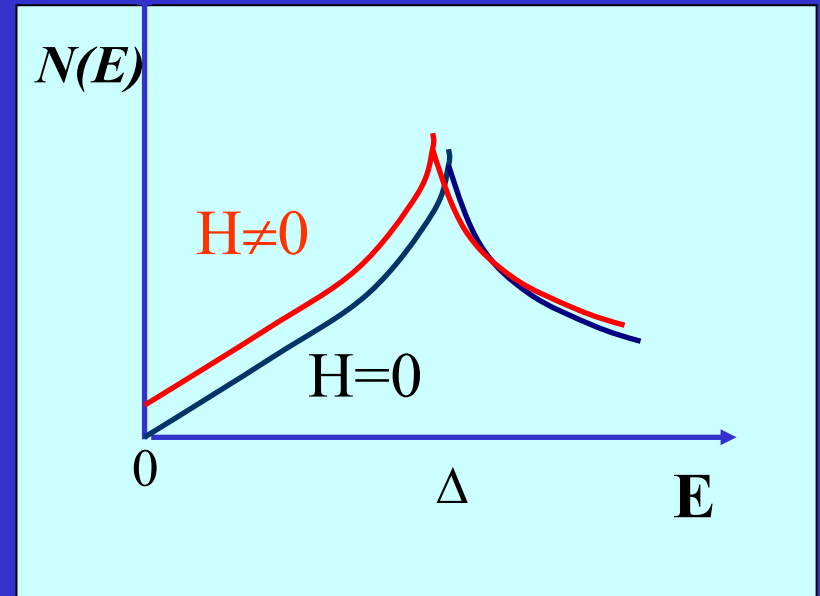
$$C_{S-wave} \approx a \gamma_n \frac{H}{H_{c2}} T$$

2. D-wave superconductor:

$$N(E_F)_{S.V.} = \int \frac{d^3 k}{(2\pi)^3} \int d^2 r \delta[E(\vec{k}, \vec{r}) - \vec{k} \cdot \vec{v}_s(\vec{r})]$$

$$\propto 1/\sqrt{H} \text{ Quasiparticle outside core}$$

$$C_{Volovik} = k \gamma_n T \sqrt{H/H_{c2}}$$



$$E_H = v_F / a_0 \propto \sqrt{H}$$

Doppler effect:

$$\delta E = \vec{v}_s \cdot \vec{k}$$

G. E. Volovik, JETP Lett. 58, 469(1993).

By solving the BdG theory for a d-wave superconductor, Simon and Lee predict a scaling law for the quasiparticle spectrum for d-wave superconductor:

S. H. Simon, P. A. Lee, PRL78, 1548 (1997).

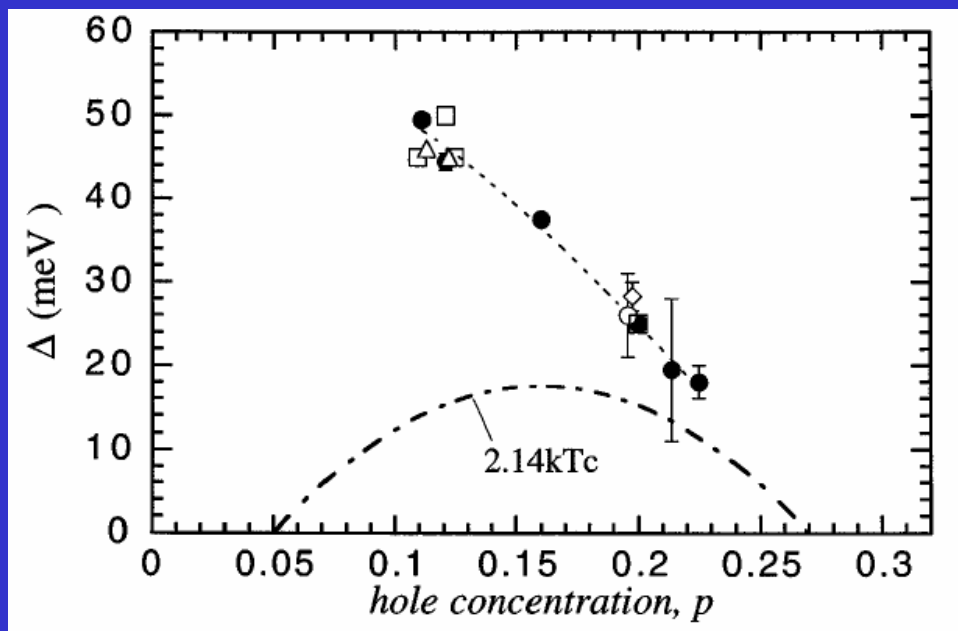
$$\begin{aligned} U &= \sum_n \epsilon_n^H f(\epsilon_n^H / T) \\ &= [H/H_0]^{1/2} \sum_n \epsilon_n^{H_0} f(\epsilon_n^{H_0} [H/H_0]^{1/2} / T), \end{aligned}$$

$$C_{vol} = T^2 f(T / \sqrt{H}) = Hg(T / \sqrt{H})$$

This scaling law contains cases in two limits: Volovik's relation $TH^{0.5}$ in low temperature limit, T^2 relation in high temperature limit. *G.E. Volovik, N. B. Kopnin, PRL78, 5028(1997).*

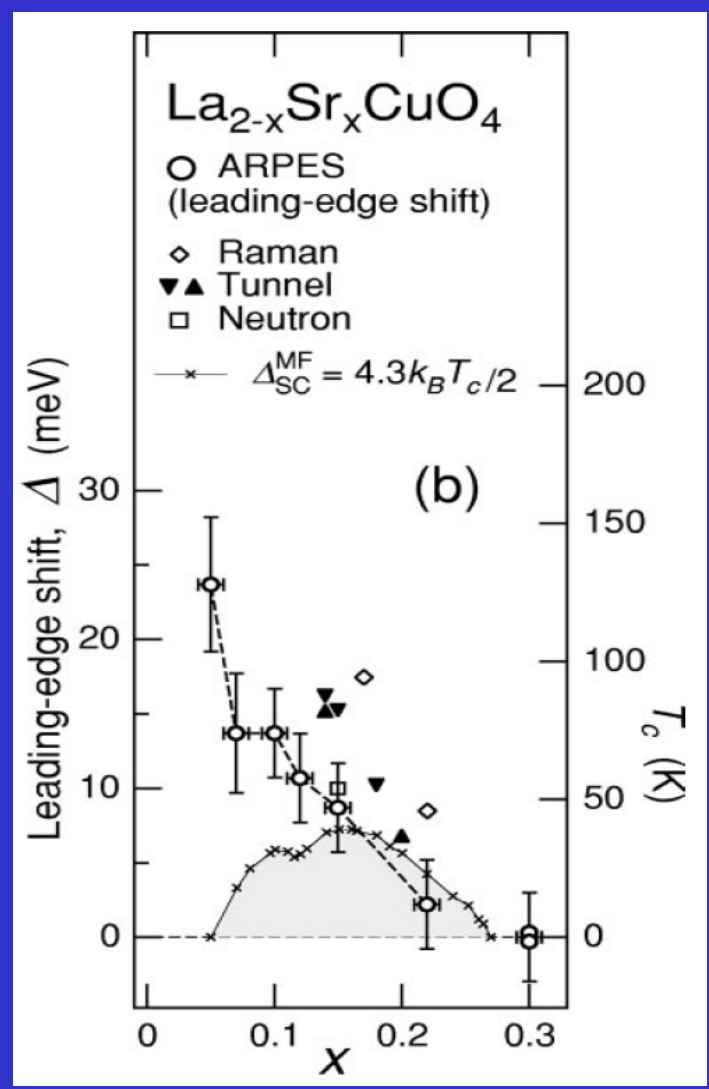
**Weak-coupling d-wave BCS
superconductivity in overdoped region**

Overdoping:



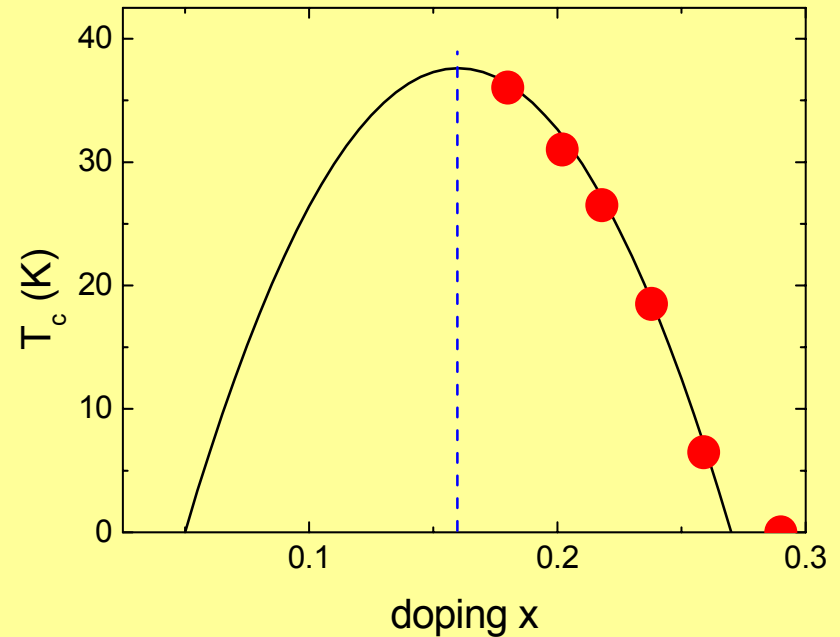
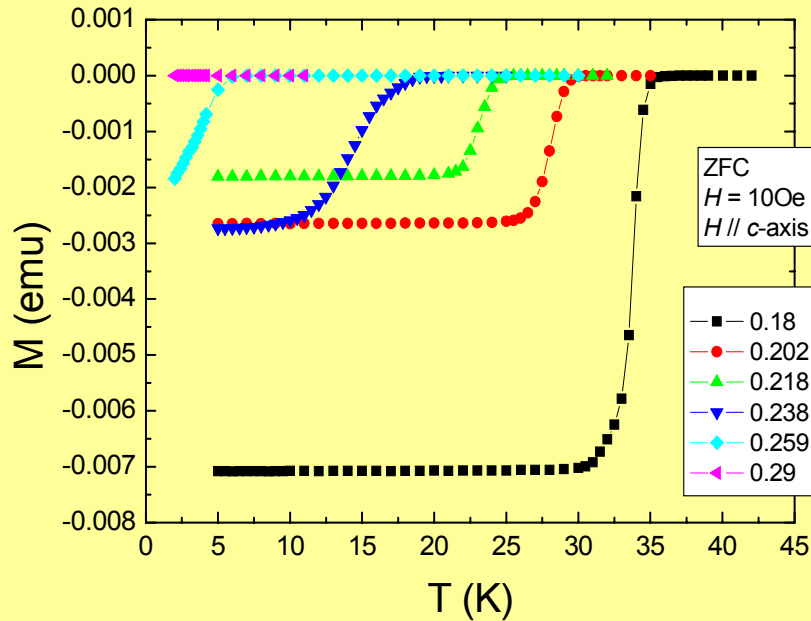
N. Miyakawa, et al., *PRL* 80, 157 (1998).

Break junction tunneling



A. Ino, et al., (2002)

Samples: $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ single crystals From Tohoku University



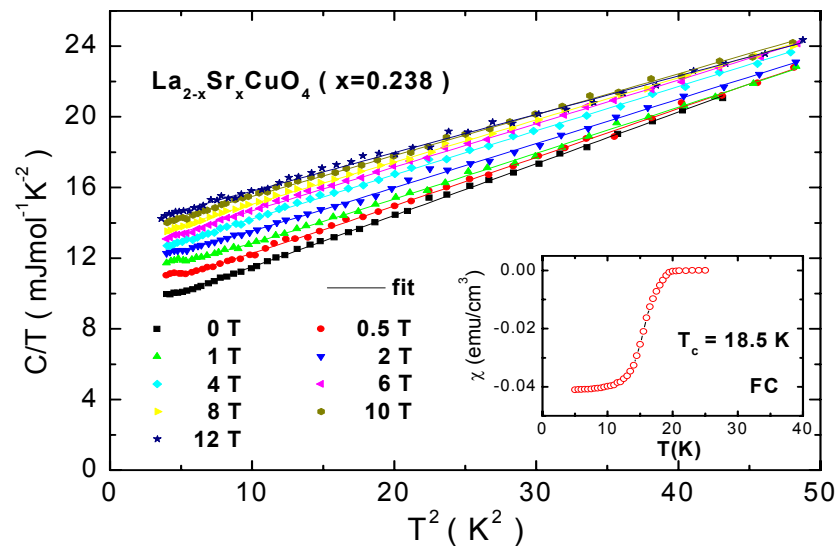
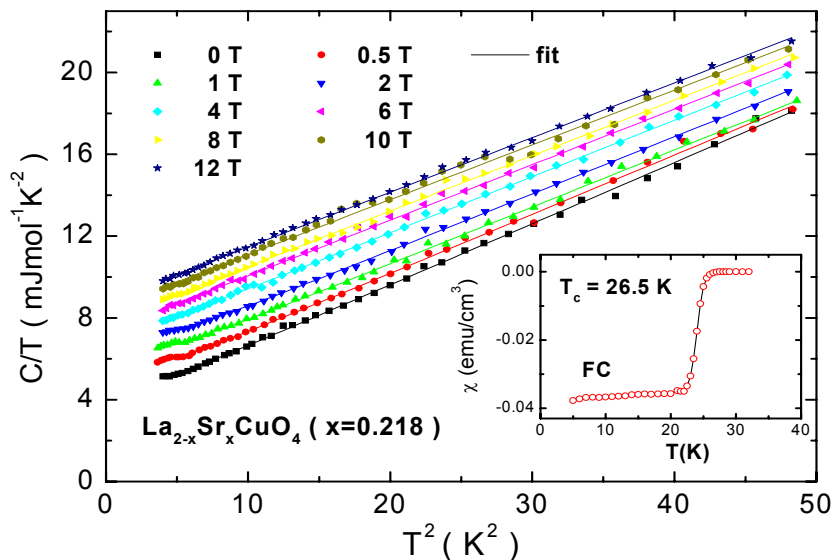
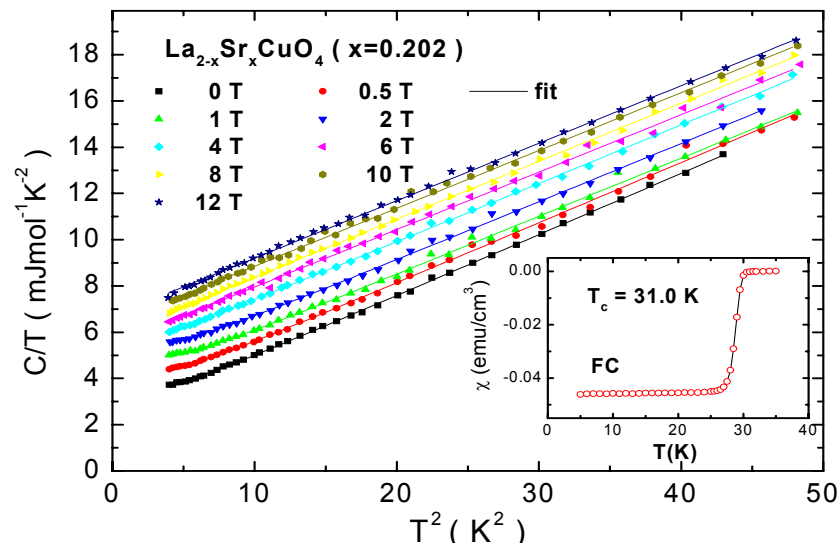
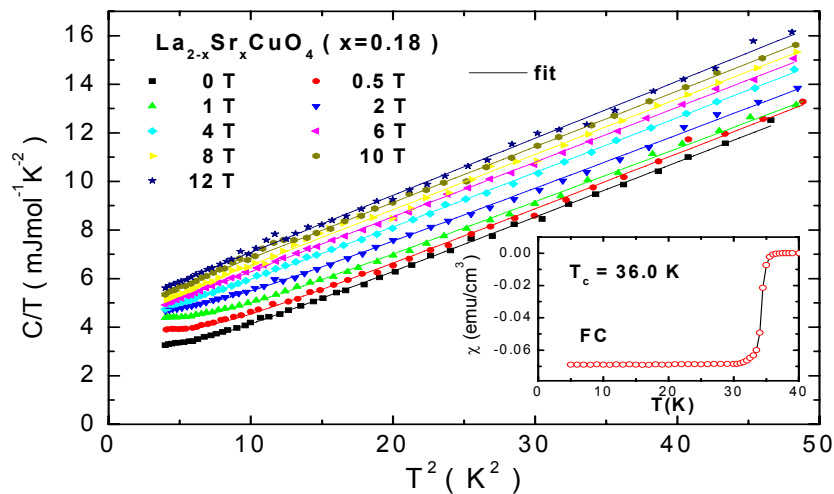
Six samples:

$x = 0.18, 0.202, 0.218, 0.238, 0.259, 0.29$

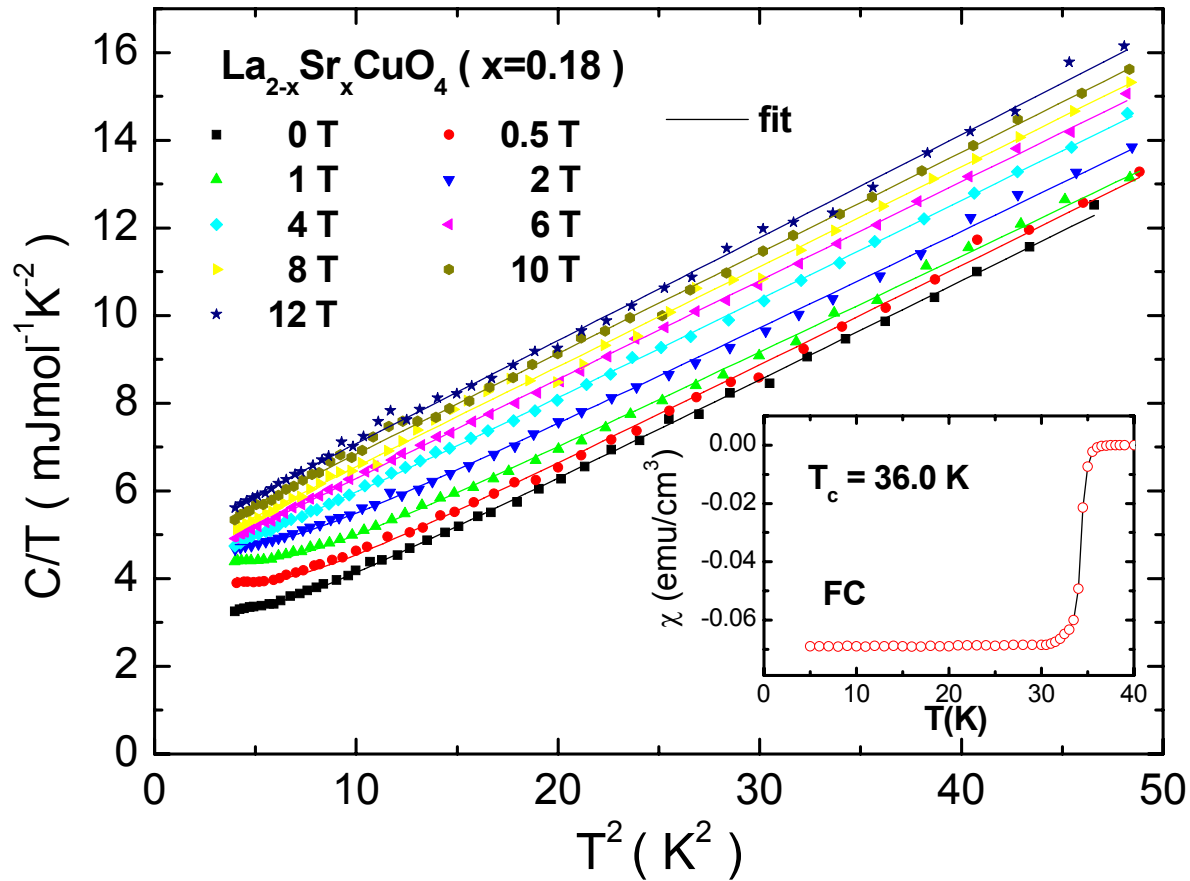
$T_c = 36, 30.5, 25, 19.5, 6.5$ and 0 K

$x=0.18\sim 0.238$

Relaxation Method



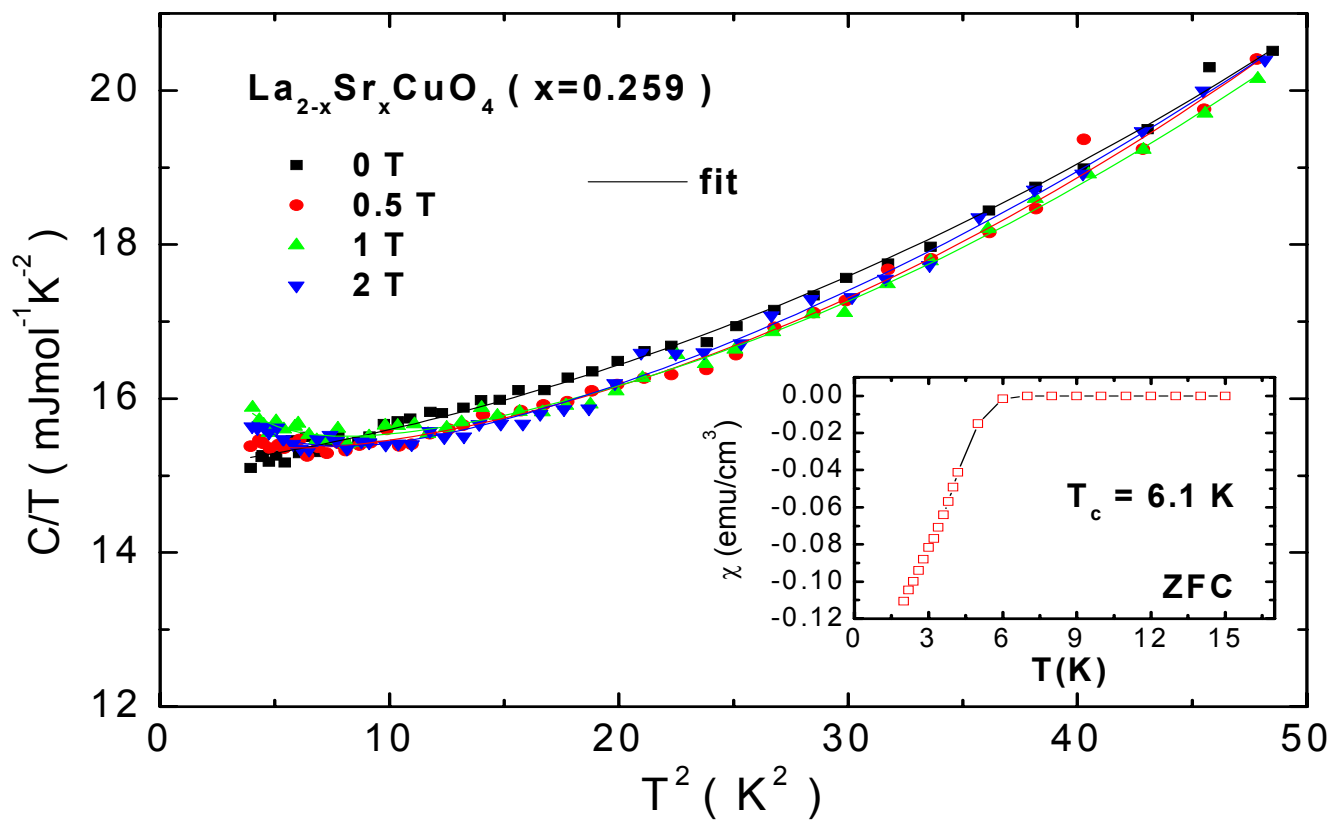
$x=0.18$



$$\frac{C}{T} = \gamma + \beta T^2 + \frac{nC_{schotky}}{T}$$

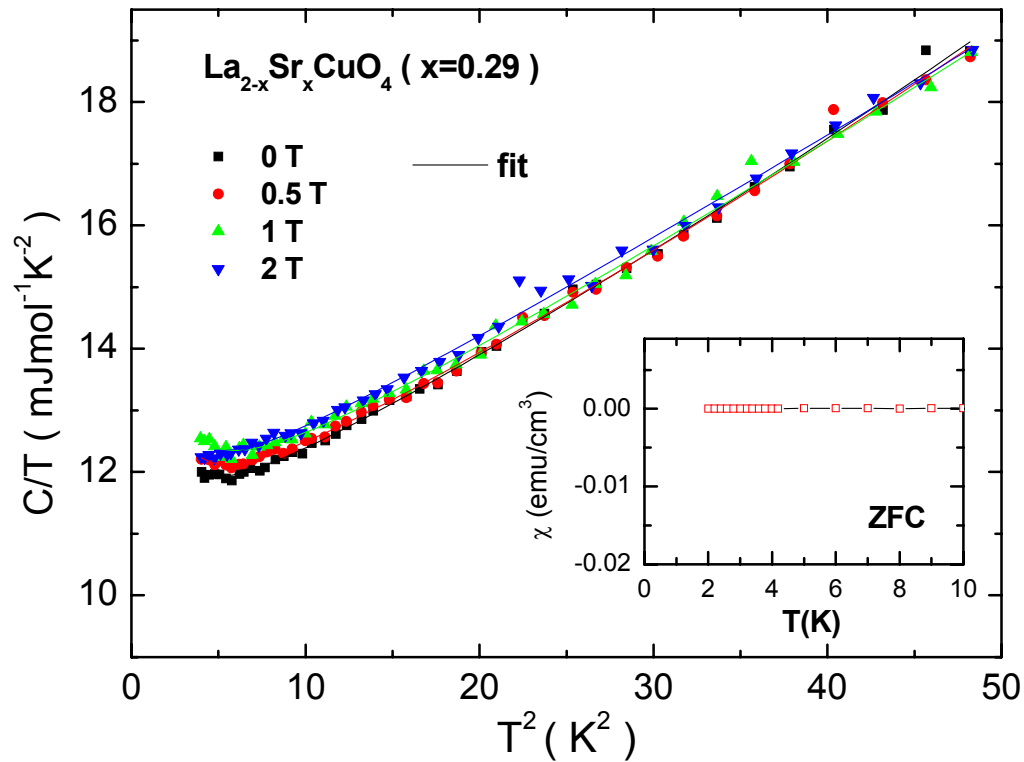
x=0.259

$$\frac{C}{T} = \gamma + \beta T^2 + \beta_5 T^4 + \frac{nC_{schottky}}{T}$$



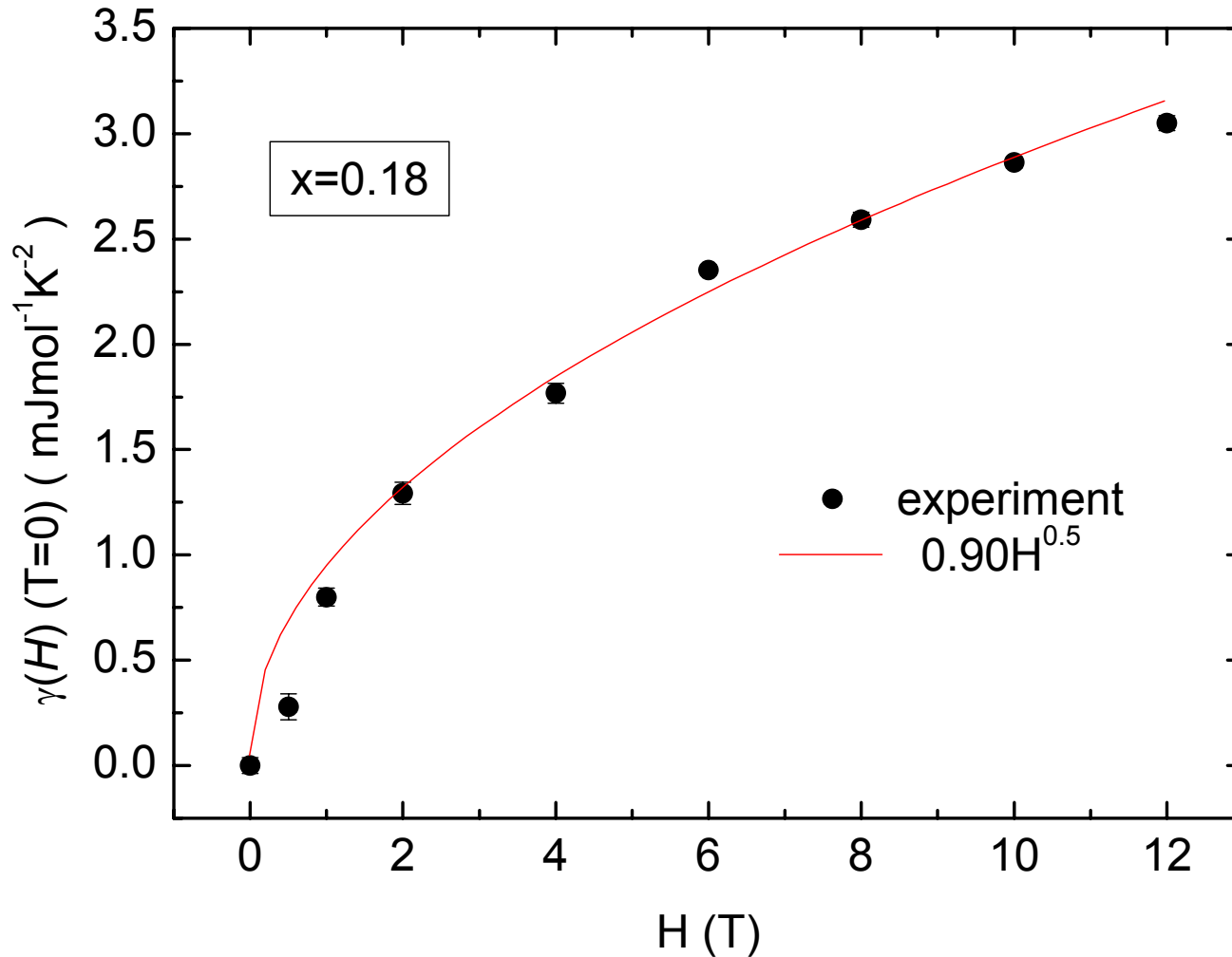
$$\frac{C}{T} = \gamma + \beta T^2 + \frac{nC_{schottky}}{T}$$

$x=0.29$



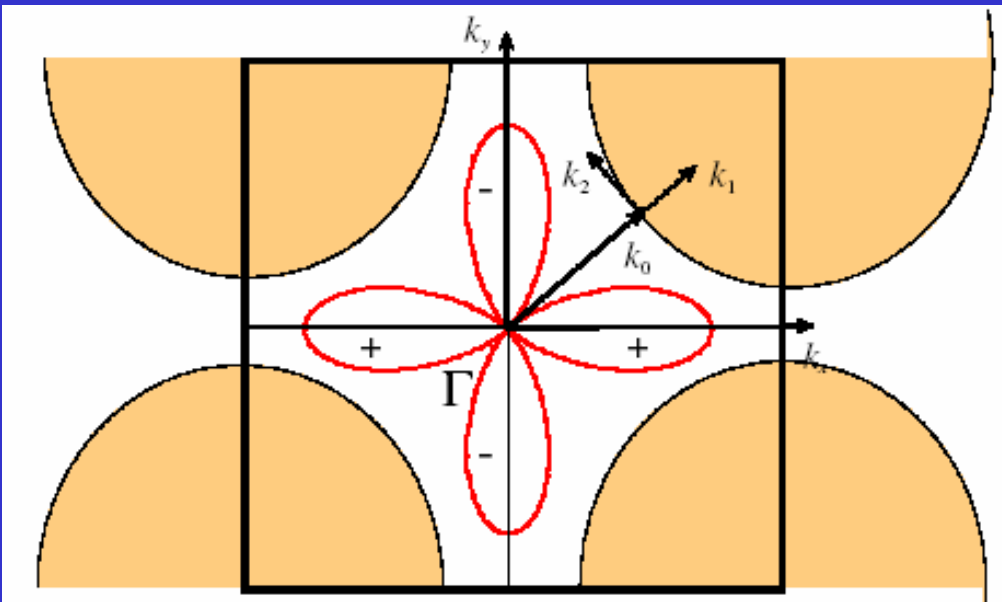
x	β	β_5	θ_D	γ_N	Notes
	($\text{mJmol}^{-1}\text{K}^{-4}$)	($\mu\text{Jmol}^{-1}\text{K}^{-6}$)	(K)	($\text{mJmol}^{-1}\text{K}^{-2}$)	
0.29	0.13	0.74	471	10.8	present work (2 K-12 K)
0.30	0.1	0.7	550	6.9	PRB 68,R100502(2003)(Nakamae <i>et al.</i>)

The field-induced specific heat for $x=0.18$:



$$\frac{C_{el}(H)}{T} = \gamma(H) = A\sqrt{H}$$

Volovik's relation
G. E. Volovik, JETP Lett. 58, 469(1993).



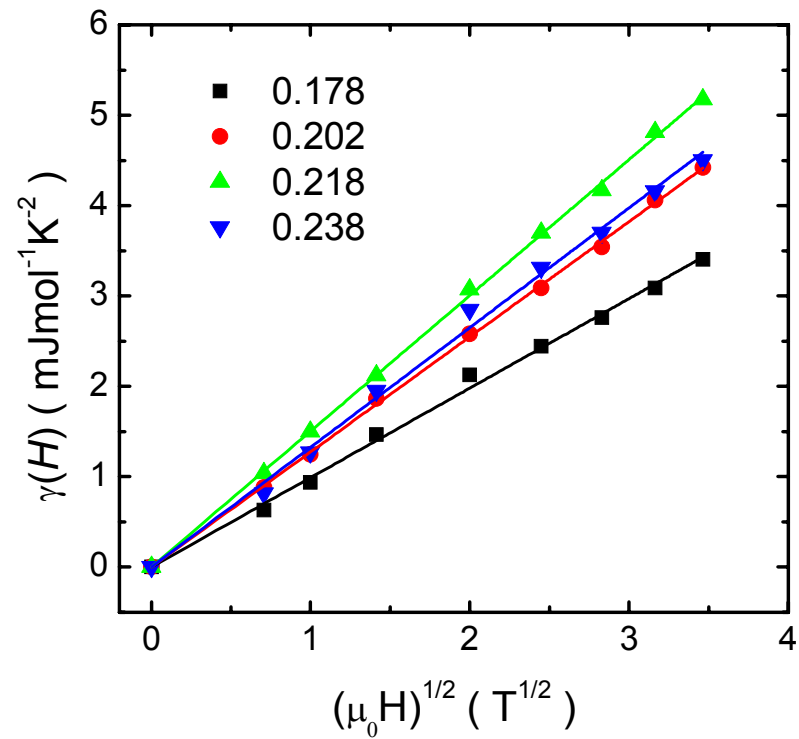
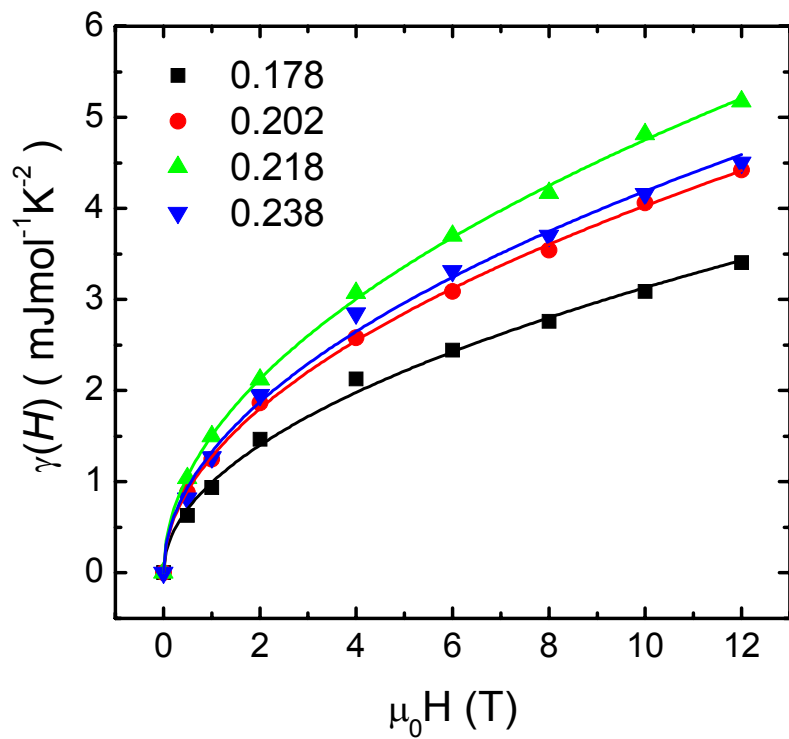
From N. E. Hussey,
Advances in Physics
51, 1685(2002).

$$\Delta(k) = \Delta_0 (\cos k_x a - \cos k_y a) = \Delta_0 \cos 2\theta$$

$$E(k) = \sqrt{\varepsilon_k^2 + \Delta_k^2} = \hbar \sqrt{v_F^2 k_{\parallel}^2 + v_{\Delta}^2 k_{\perp}^2}$$

$$N(E) = \frac{2}{\pi \hbar^2} \left(\frac{1}{v_F v_{\Delta}} \right) E \quad v_{\Delta} = \left[\frac{d\Delta_s}{d\phi} \right]_{node} / \hbar k_F$$

$$A = \frac{4k_B^2}{3\hbar} \left(\frac{\pi}{\Phi_0} \right)^{1/2} \frac{nV_{mol}}{d} \frac{a}{v_{\Delta}}$$



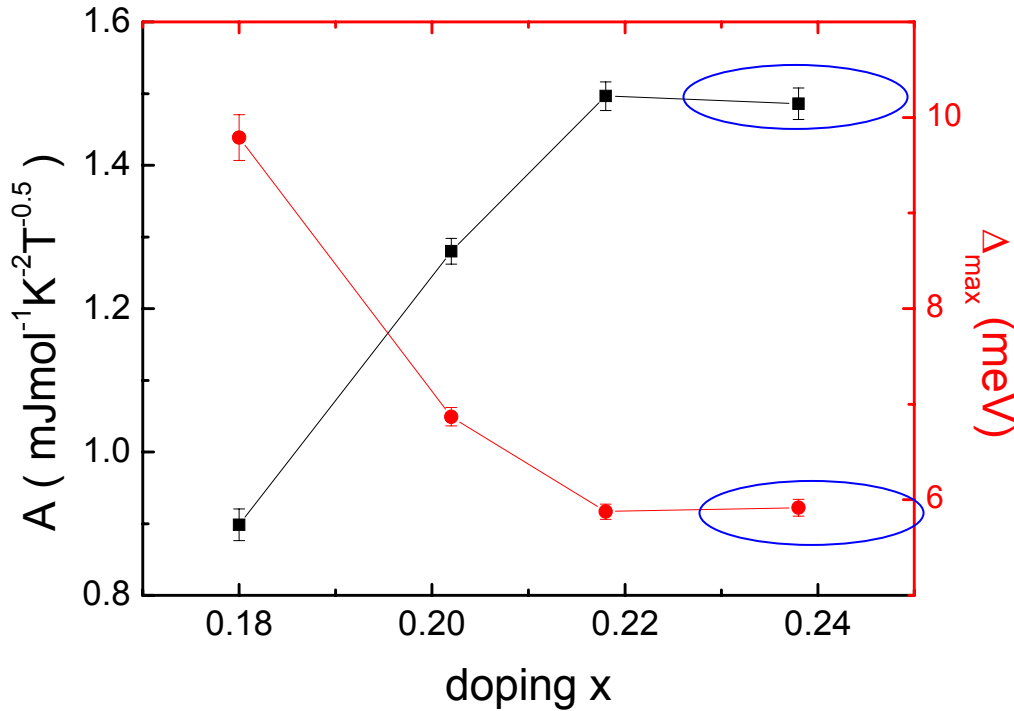
$$\frac{C_{el}(H)}{T} = \gamma(H) = A\sqrt{H}$$

Robust d -wave symmetry

$$A = \frac{4k_B^2}{3\hbar} \left(\frac{\pi}{\Phi_0} \right)^{1/2} \frac{nV_{mol}}{d} \frac{a}{v_\Delta}$$

$$\Delta_0 = \frac{1}{2} \hbar k_F v_\Delta$$

$$A = \frac{Const}{\Delta_{max}}$$

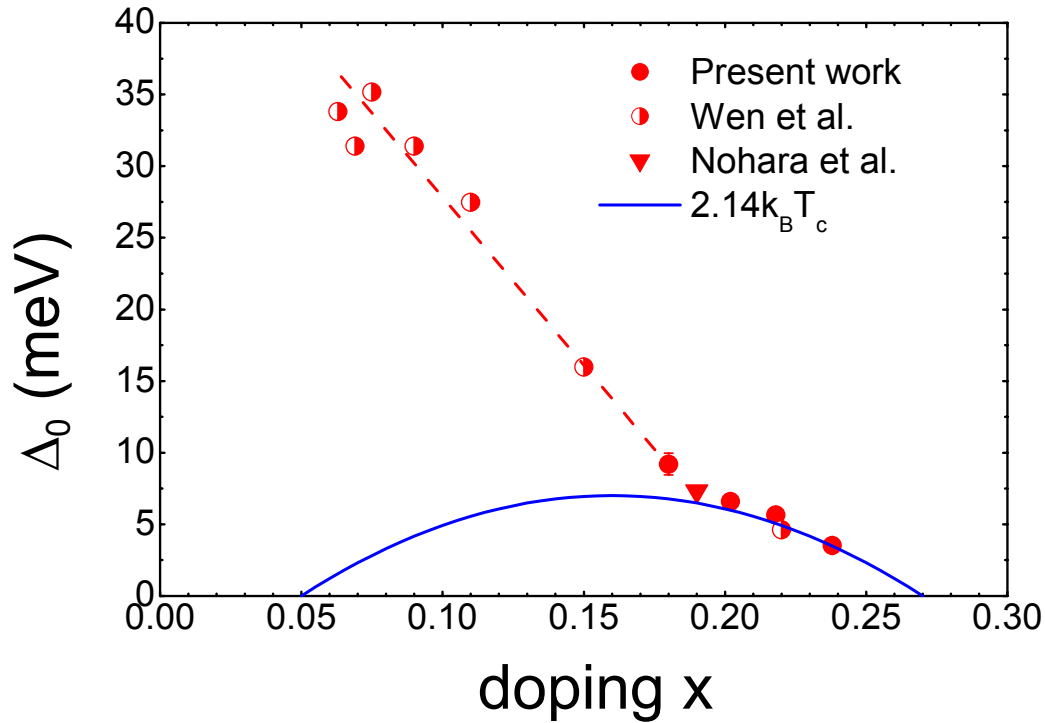


No longer increase in A: due to the decrease of the superconducting volume fraction

$$A_{corr} = A \gamma_N / [\gamma_N - \gamma(0)]$$

Correction needed!

Weak coupling d-wave superconductivity in overdoped region!



$$A = \frac{4k_B^2}{3\hbar} \left(\frac{\pi}{\Phi_0} \right)^{1/2} \frac{nV_{mol}}{d} \frac{a}{v_\Delta}$$

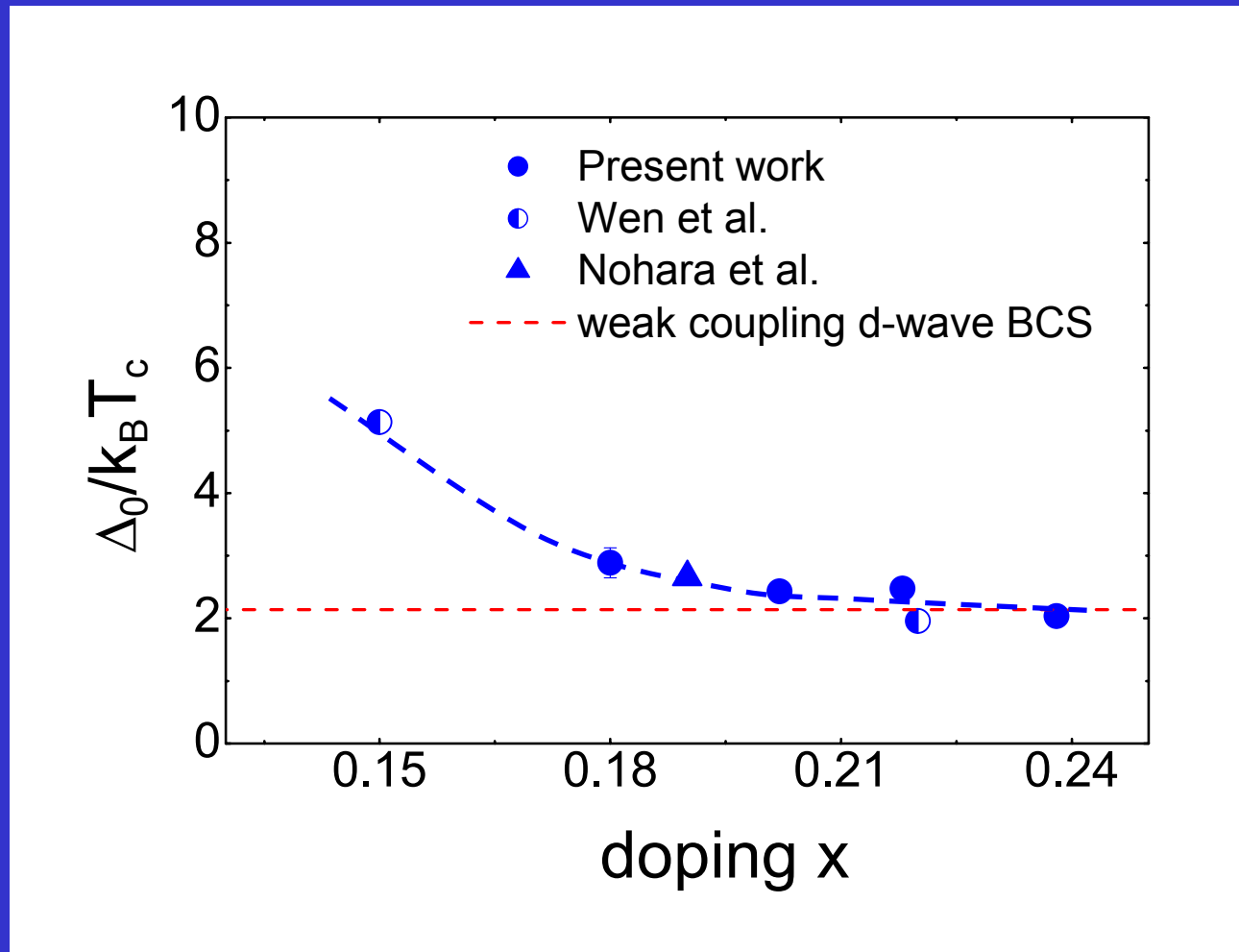
$$\Delta_0 = \frac{1}{2} \hbar k_F v_\Delta$$

Underdoped LSCO:

H. H. Wen, et al., PRB72, 134507(2005).

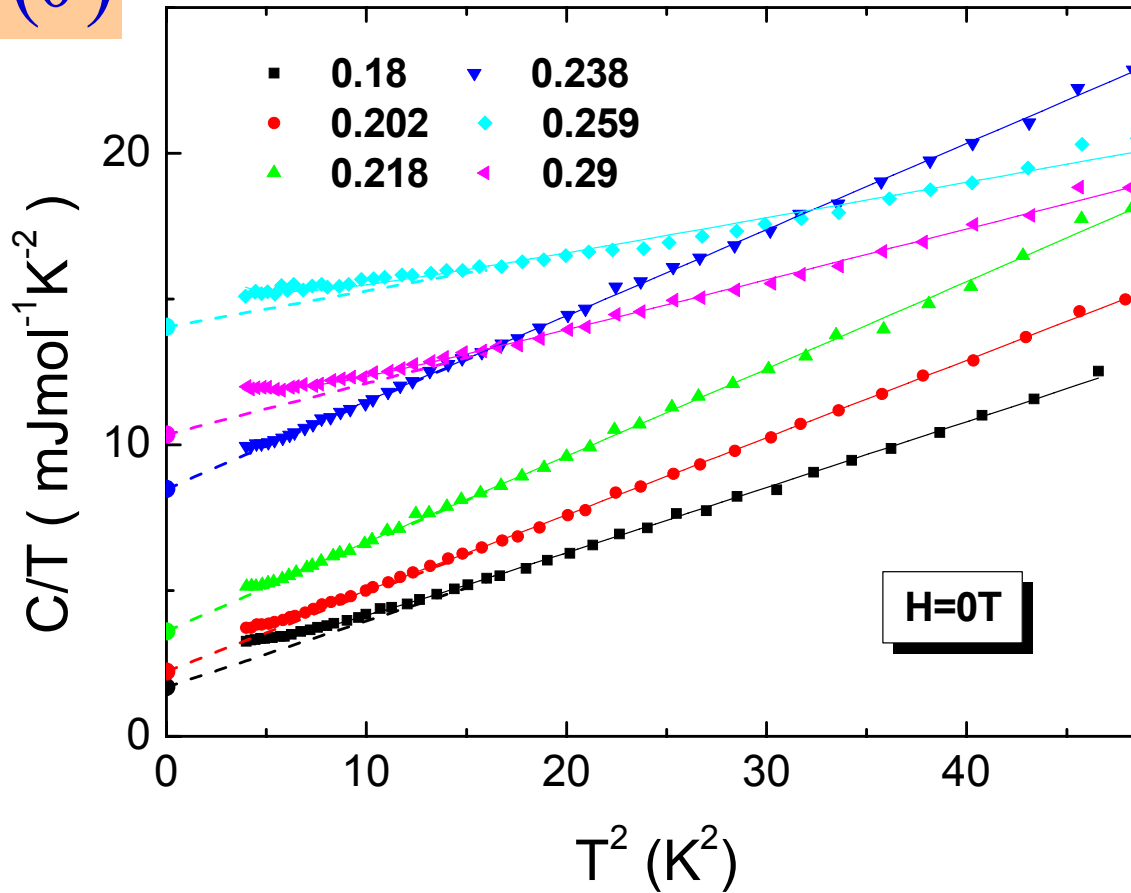
H. H. Wen, et al., PR B70, 214505(2004).

Weak coupling d-wave superconductivity in overdoped region!

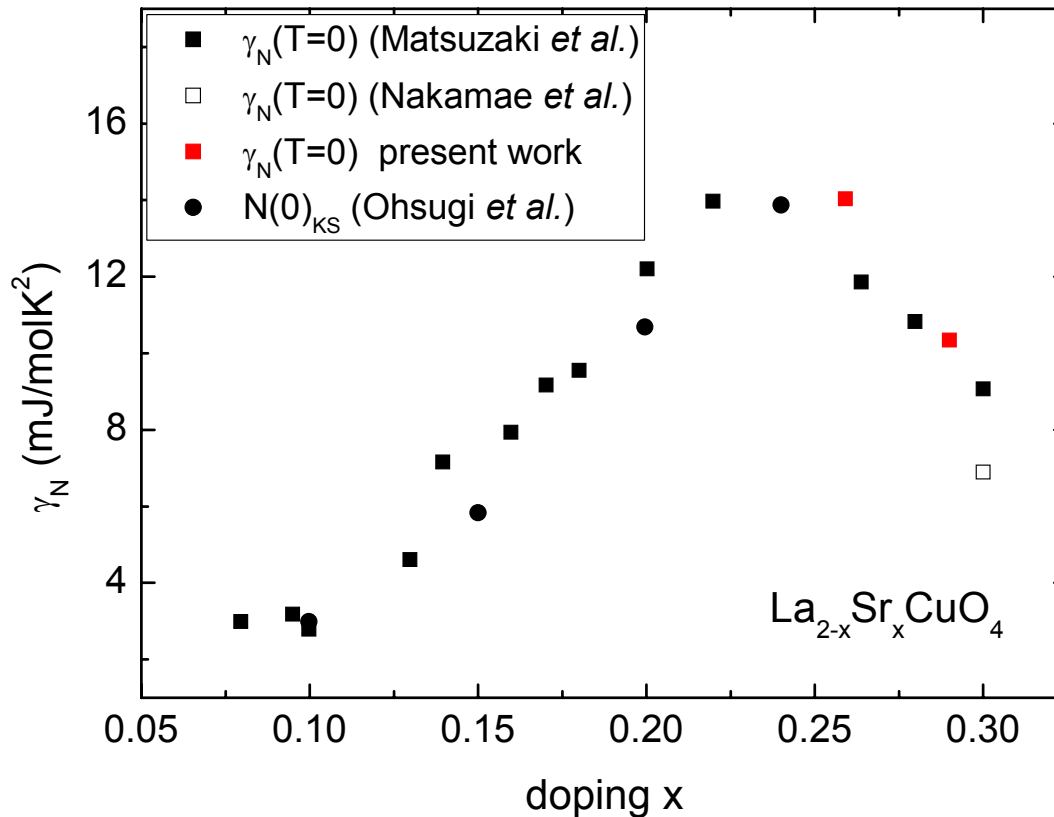


Yue Wang, et al., Phys. Rev. B, 2007, in press.

$\gamma(0)$



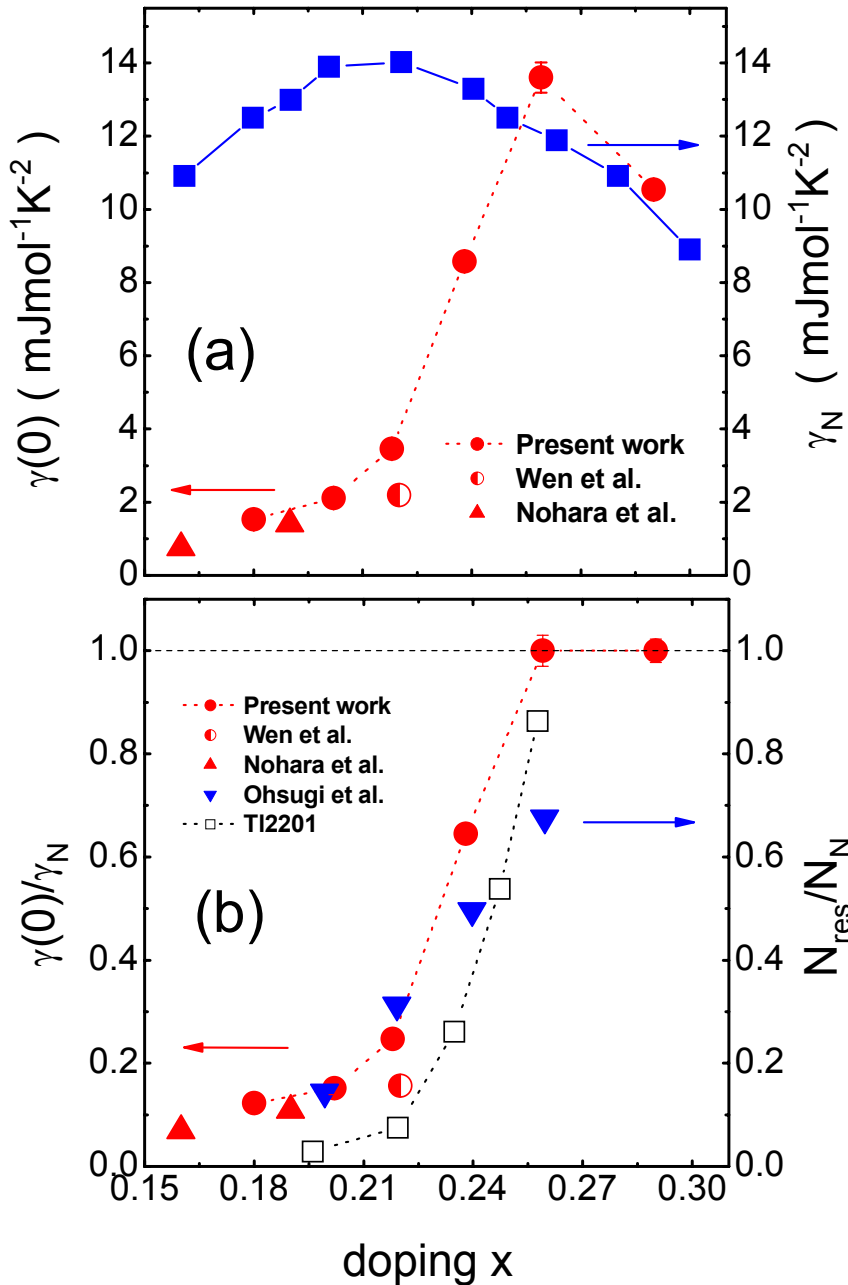
$\gamma(0)$ increases rapidly with increasing doping ($0.18 \leq x \leq 0.259$) and evolves into the normal state γ_N at $x = 0.29$



S. Nakamae *et al.*, PRB 68, R100205 (2003)

T. Matsuzaki *et al.*, J. Phys. Soc. Jpn. 73, 2232 (2004)

Crudely estimate the fraction of the residual unpaired carries:



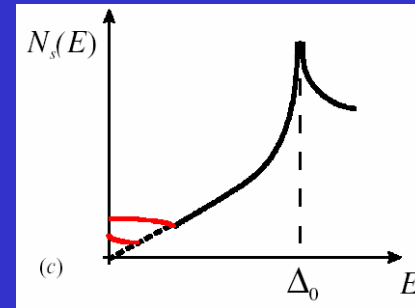
x	γ_N	$\gamma(0)$	$\frac{\gamma(0)}{\gamma_N}$
	(mJmol ⁻¹ K ⁻²)	(mJmol ⁻¹ K ⁻²)	(%)
0.18	9.6	1.68	17.5
0.202	12.2	2.22	18.2
0.218	13.9	3.60	25.9
0.238	13.9	8.48	61.0
0.259	~14.03	14.03	~100
0.29	10.34	10.34	100

Growing of the normal state regions in the sample towards more overdoping

Impurity scattering:

$$\frac{\gamma(H)}{\gamma_N} = \frac{\Delta_0}{8\gamma_0} a^2 \left(\frac{H}{H_{c2}} \right) \ln \left[\frac{\pi}{2a^2} \left(\frac{H_{c2}}{H} \right) \right]$$

$$\frac{\gamma_{res}^i}{\gamma_N} = \frac{2\gamma_0}{\pi\Delta_0} \ln \left(\frac{\Delta_0}{\gamma_0} \right)$$

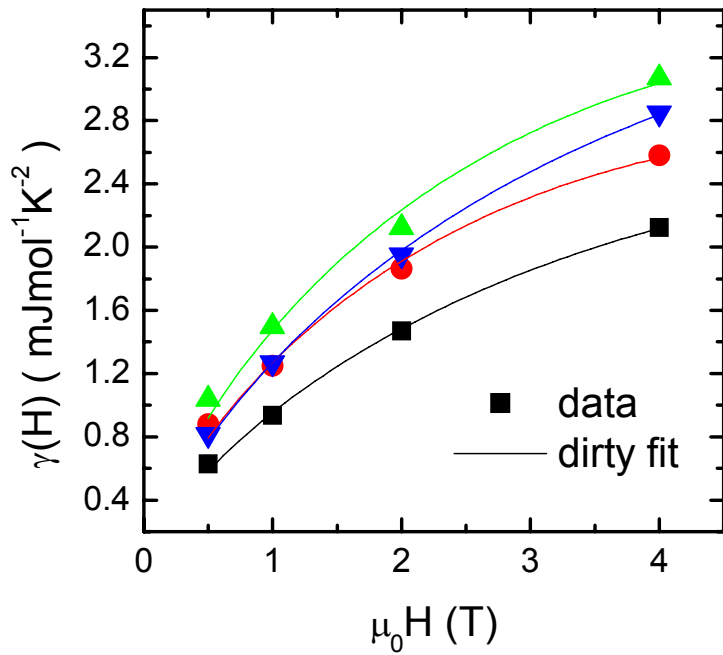


$$\gamma_0 \cong 0.61 \sqrt{\Gamma \Delta_0} \quad \text{pair-breaking parameter}$$

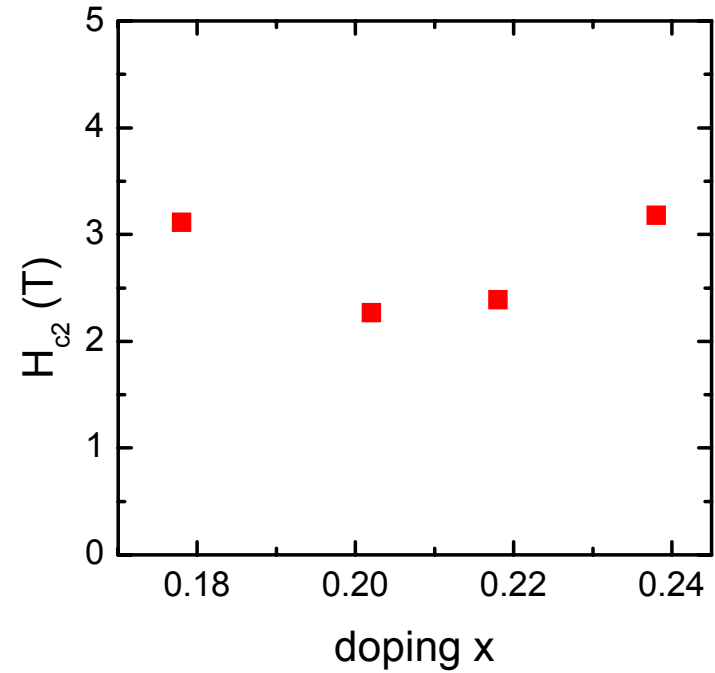
$$\Gamma = \frac{n_{imp}}{\hbar \pi N_n(0)} \quad \text{quasi-particle scattering rate}$$

C. Kübert et al., Solid State Commun. 105, 459(1998)

G. Preosti et al., PRB 50, 1259 (1994)



dirty fit



derived H_{c2} extremely small,
unreasonable!

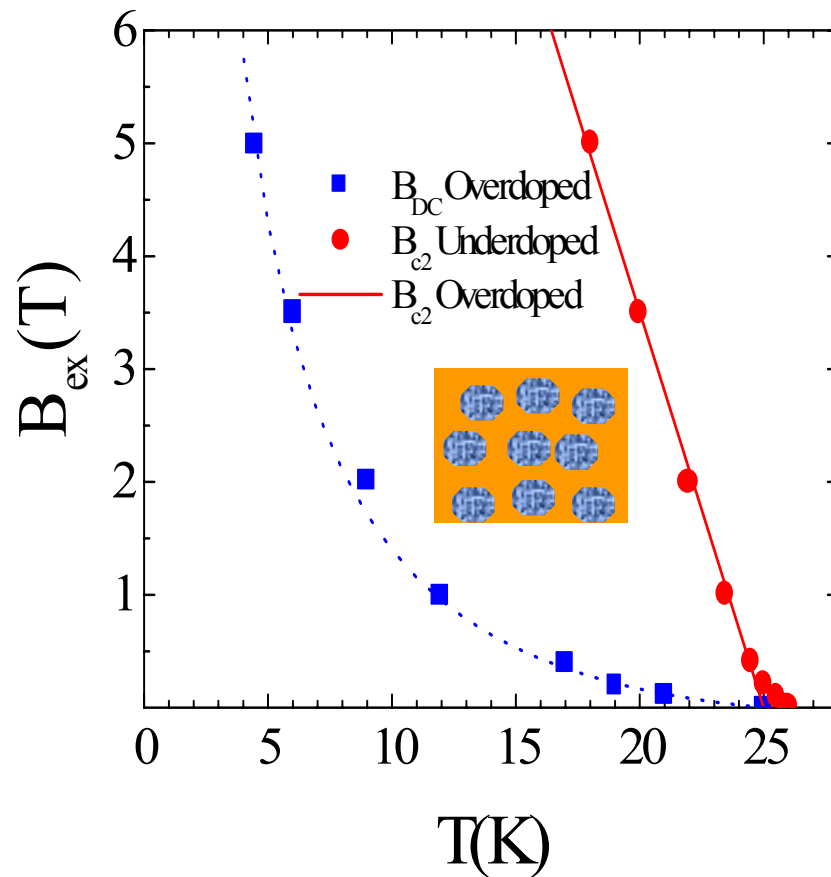
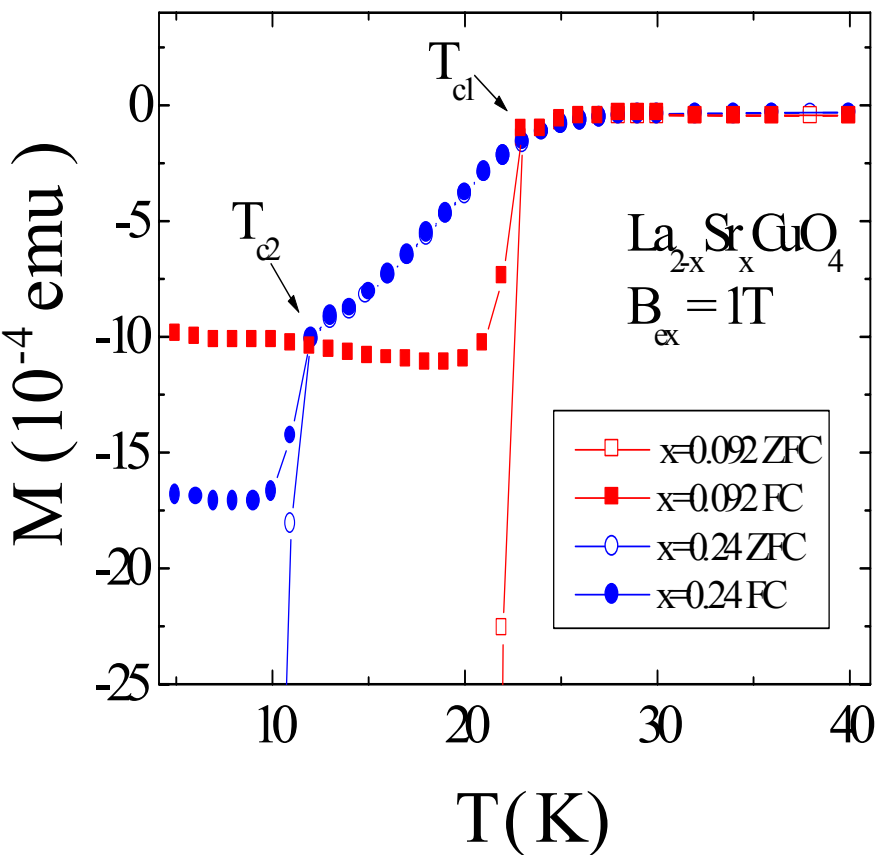
$$\frac{\gamma(H)}{\gamma_N} = \frac{\Delta_0}{8\gamma_0} a^2 \left(\frac{H}{H_{c2}} \right) \log \left[\frac{\pi}{2a^2} \left(\frac{H_{c2}}{H} \right) \right]$$

Failure of the fitting !

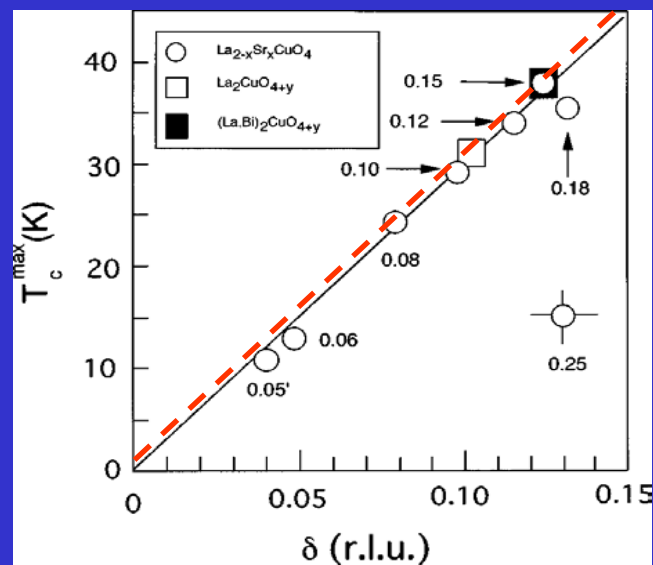
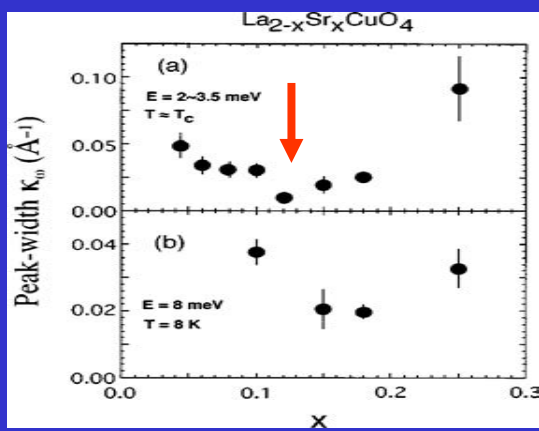
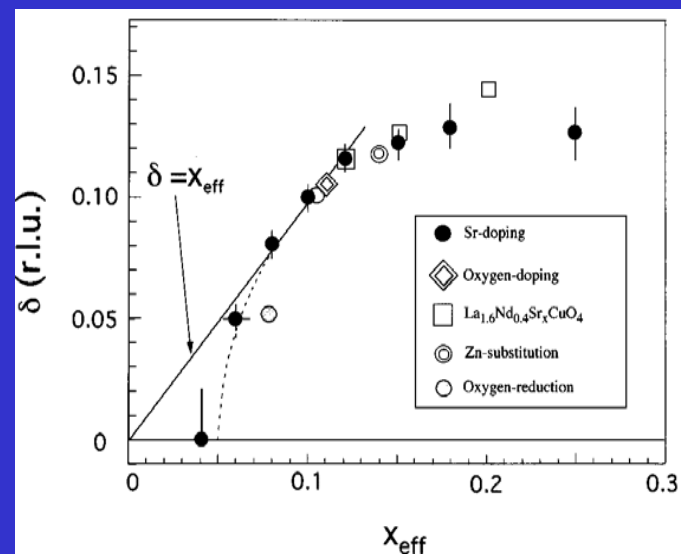
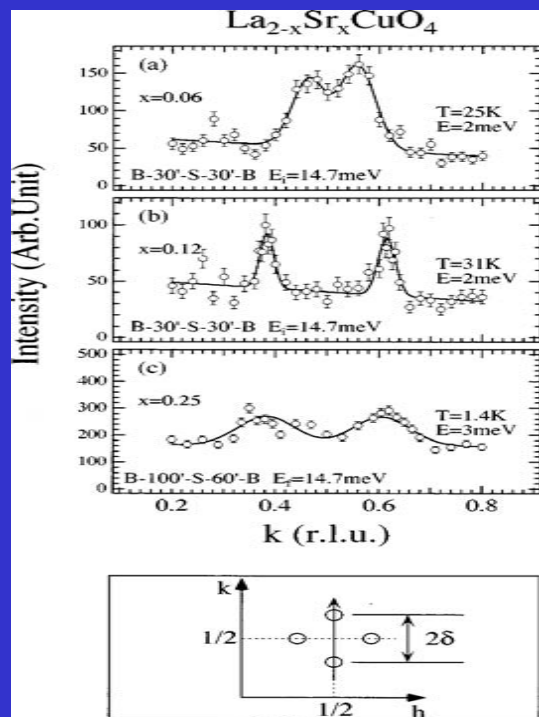
Data from $\text{La}_{2-x}\text{Sr}_x\text{CuO}_6$ single crystals

H. H. Wen, X. H. Chen, W. L. Yang, Z. X. Zhao

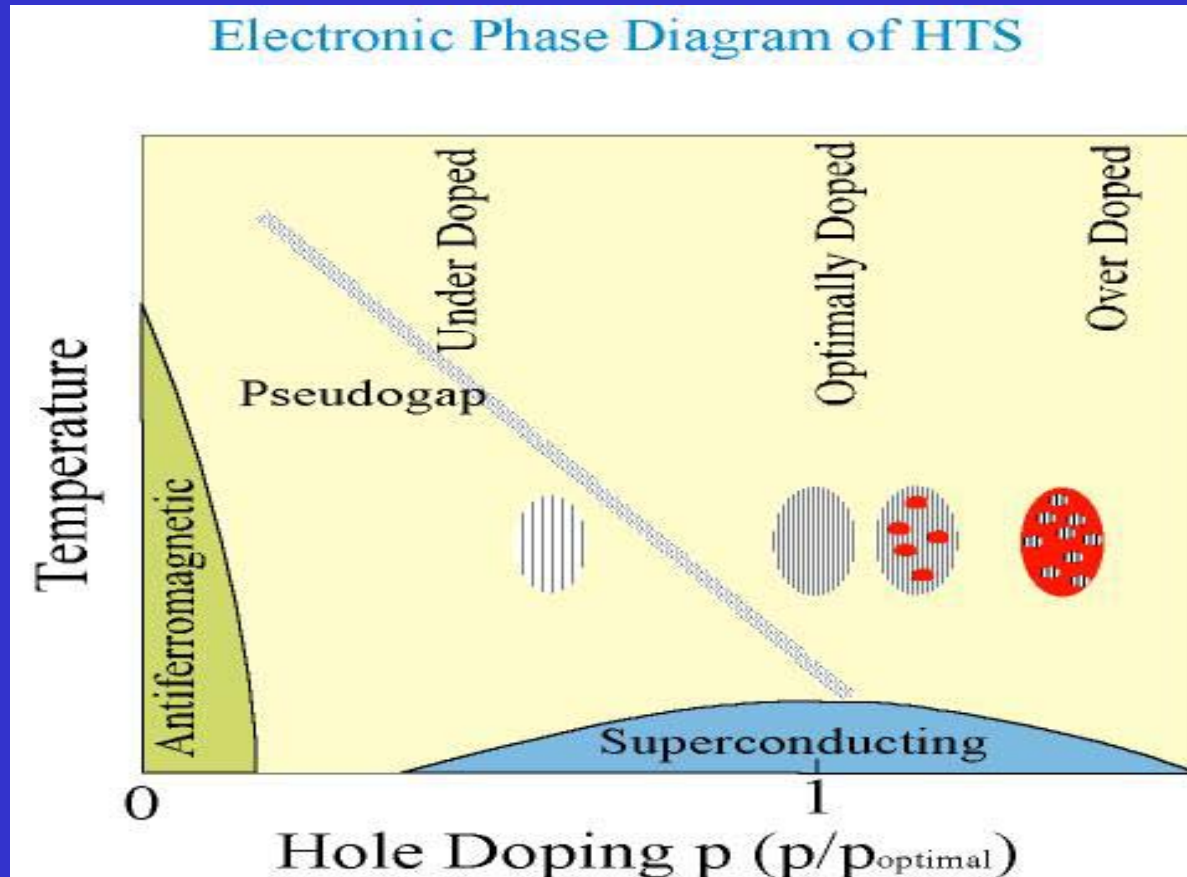
Phys. Rev. Lett. 85, 2805(2000) ; *PNAS* 97, 11145 (2000).



K. Yamada et al. PRB 57, 6165(1998).



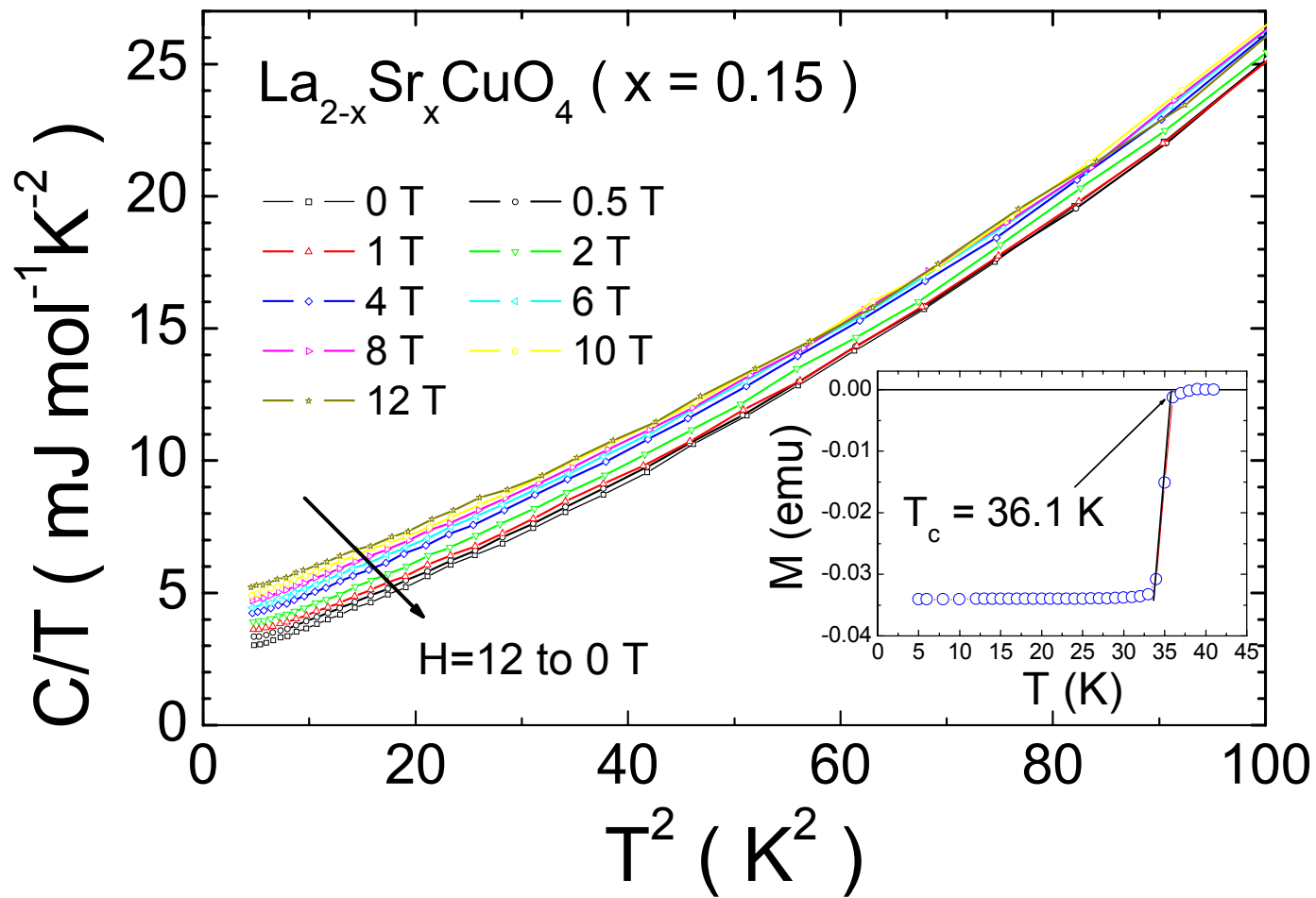
*Proposed Picture for
Electronic Phase Diagram (By Hai-Hu Wen 2000)*



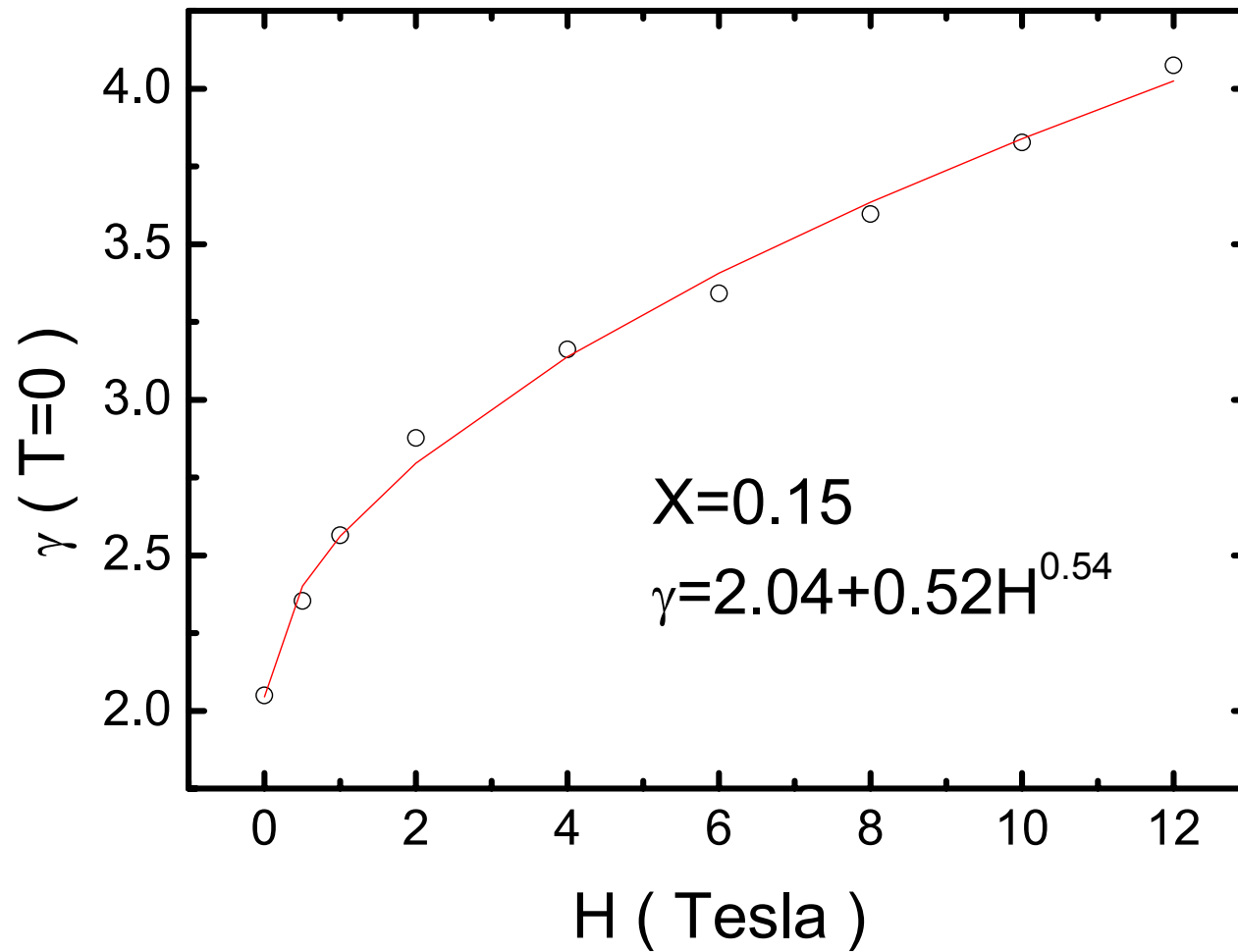
Similar conclusion: S. Wakimoto, et al., Phys. Rev. Lett. 98, 247003(2007)

Underdoped region:

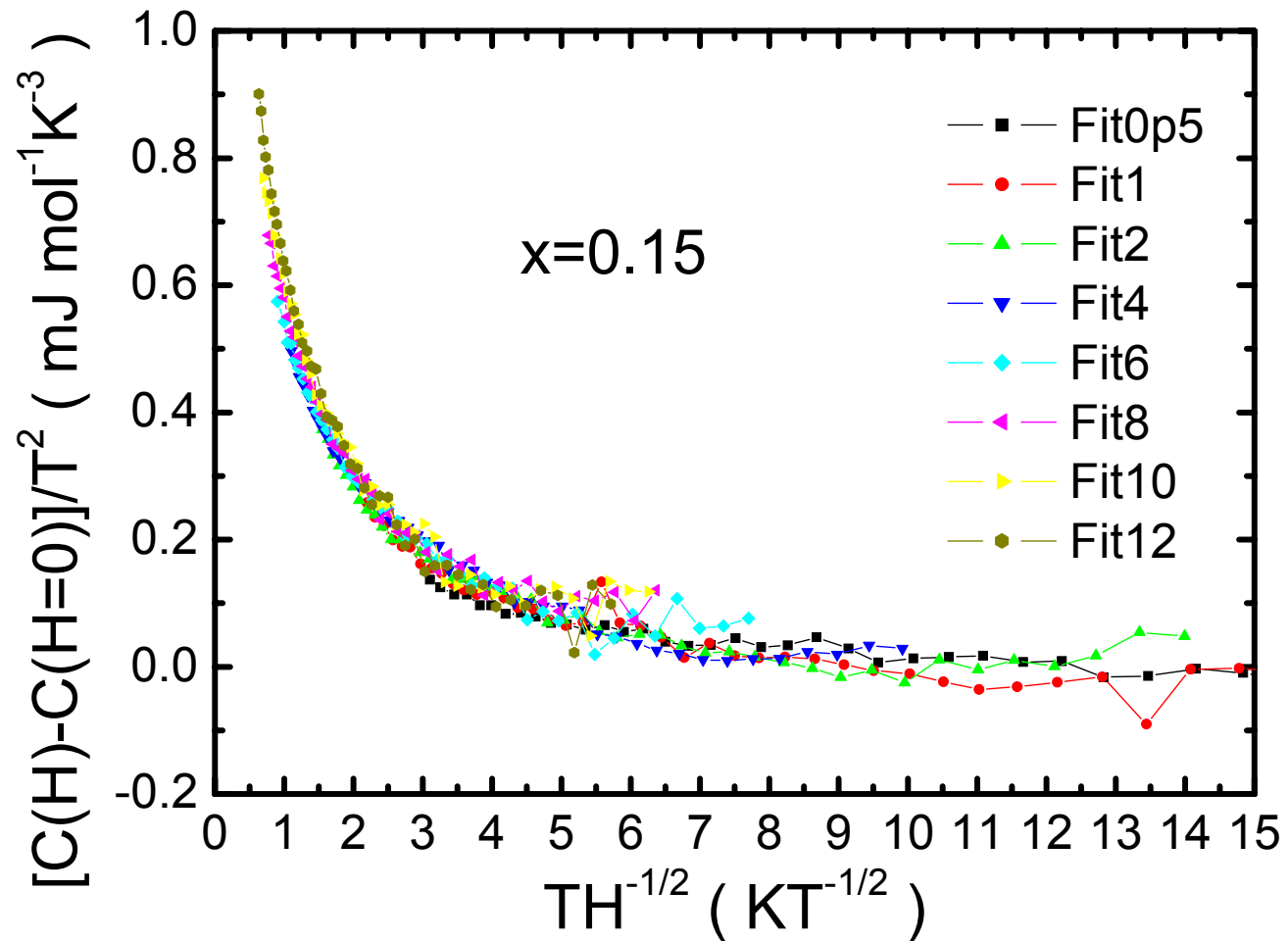
**Correlation between pseudogap
and superconductivity**



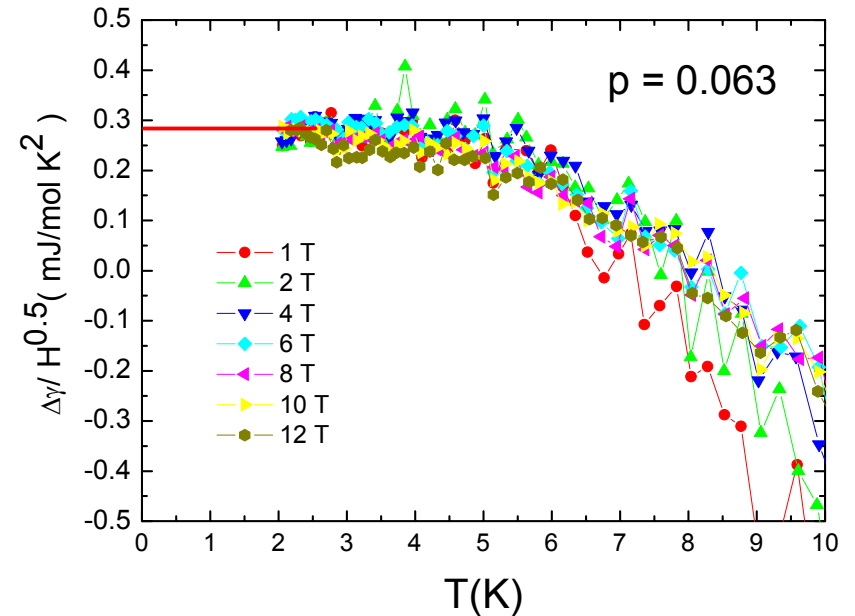
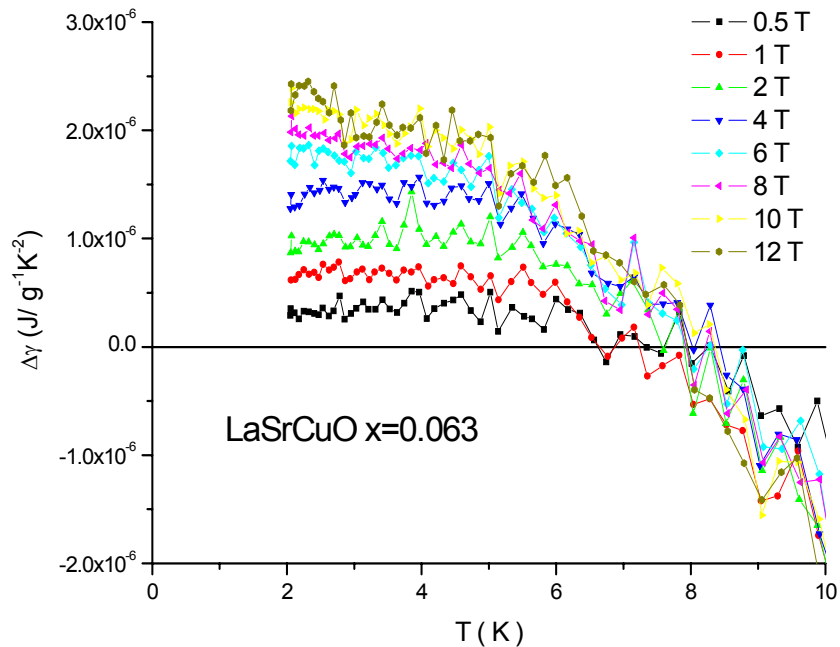
$x=0.15$



Pure d-wave scaling $x=0.15$



LaSrCuO, $p=0.063$

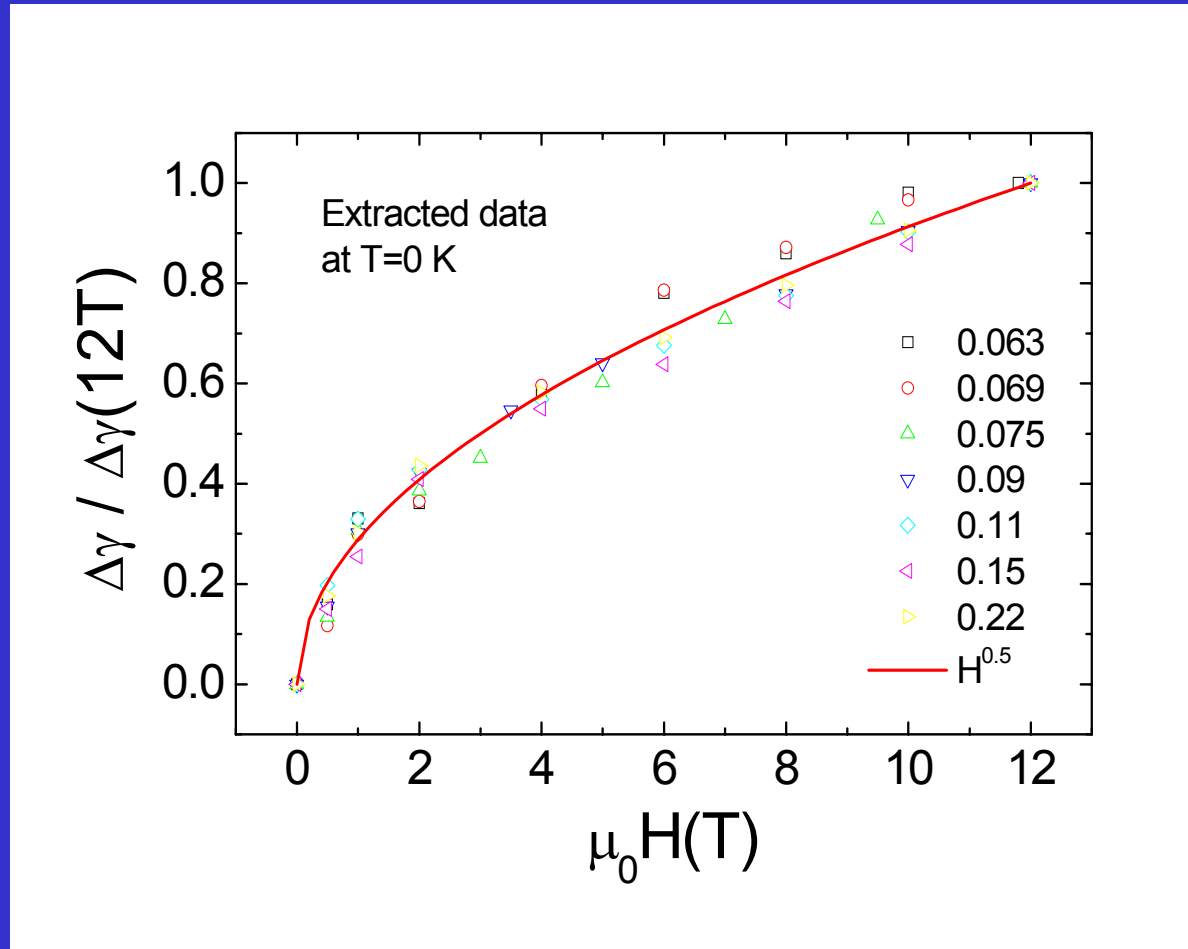


Raw data $\Delta\gamma$ vs. T

$\Delta\gamma/H^{0.5}$ vs. T

Good scaling to Volovik's relation at $T=0$ for all doping concentrations:

$$\gamma_{vol} = A\sqrt{H}$$



H. H. Wen, et al., Phys. Rev. B70, 214505(2004).

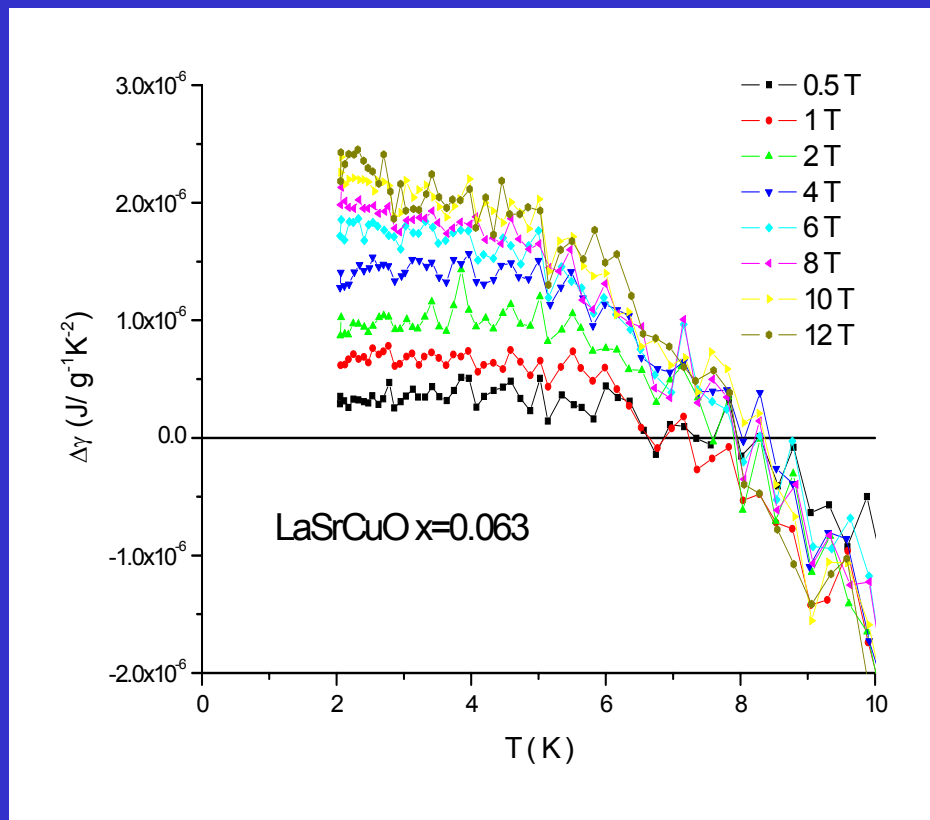
**Ground state of the
pseudogap phase:**

Nodal metal

Vs.

Fermi arc metal

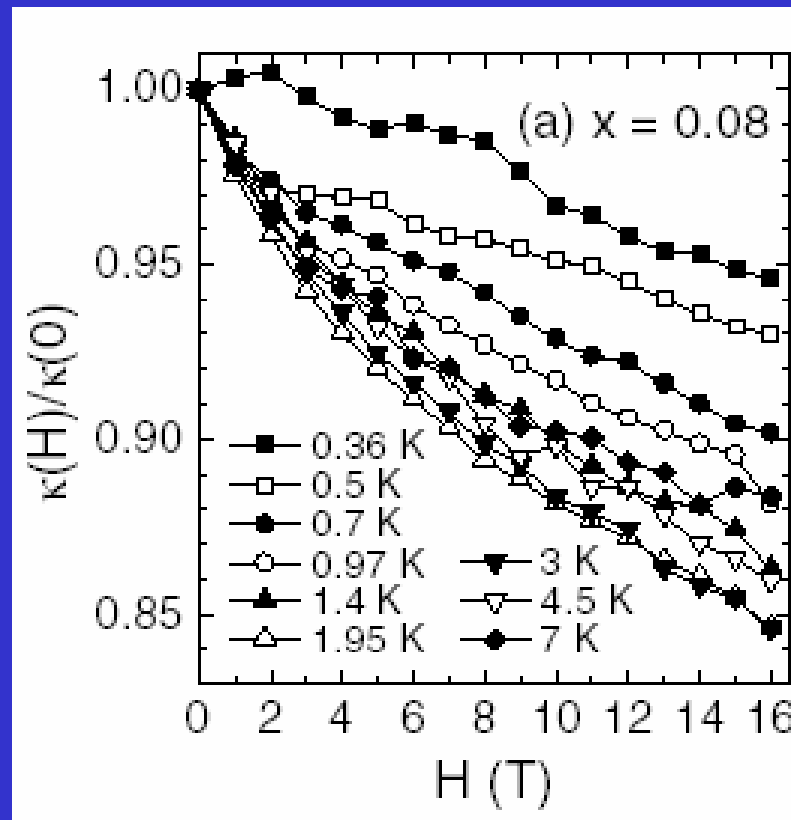
LaSrCuO, $p=0.063$



$$\gamma \propto N(E_F)$$

Our data

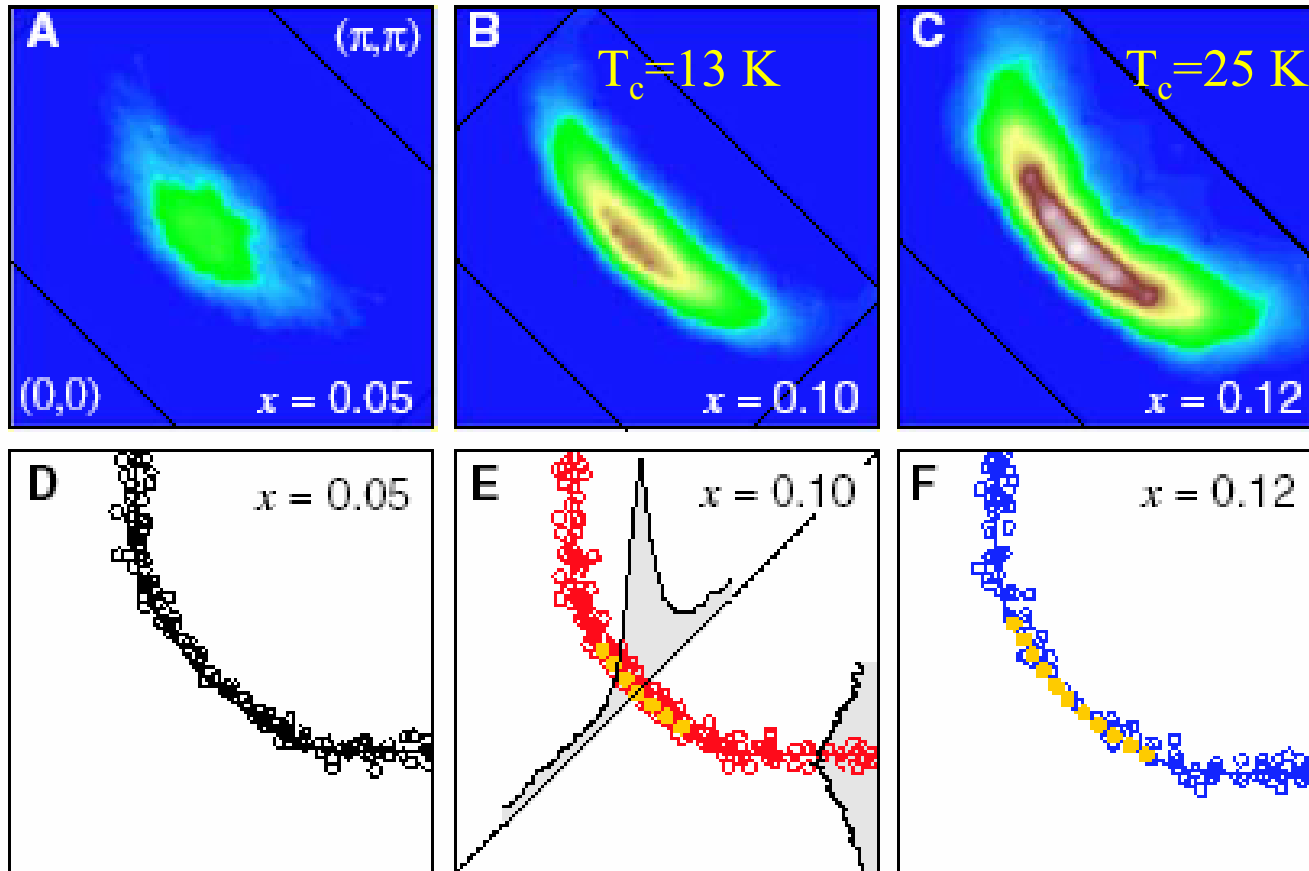
$p=0.08$



$$\kappa \propto N(E_F)\tau$$

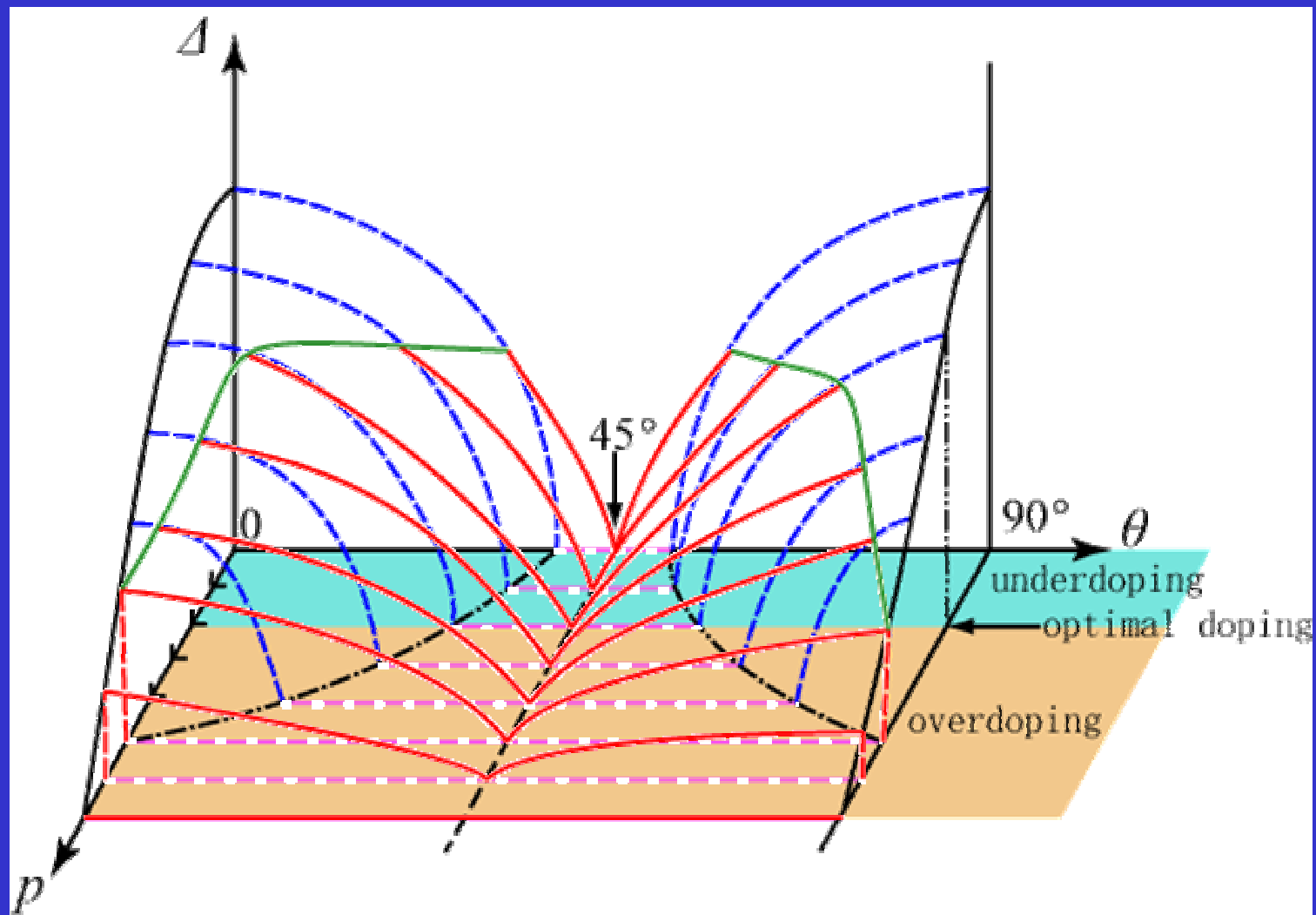
X. F. Sun, ..., Y. Ando,
PRL(2003)

Na-CaCuOCl Single crystals

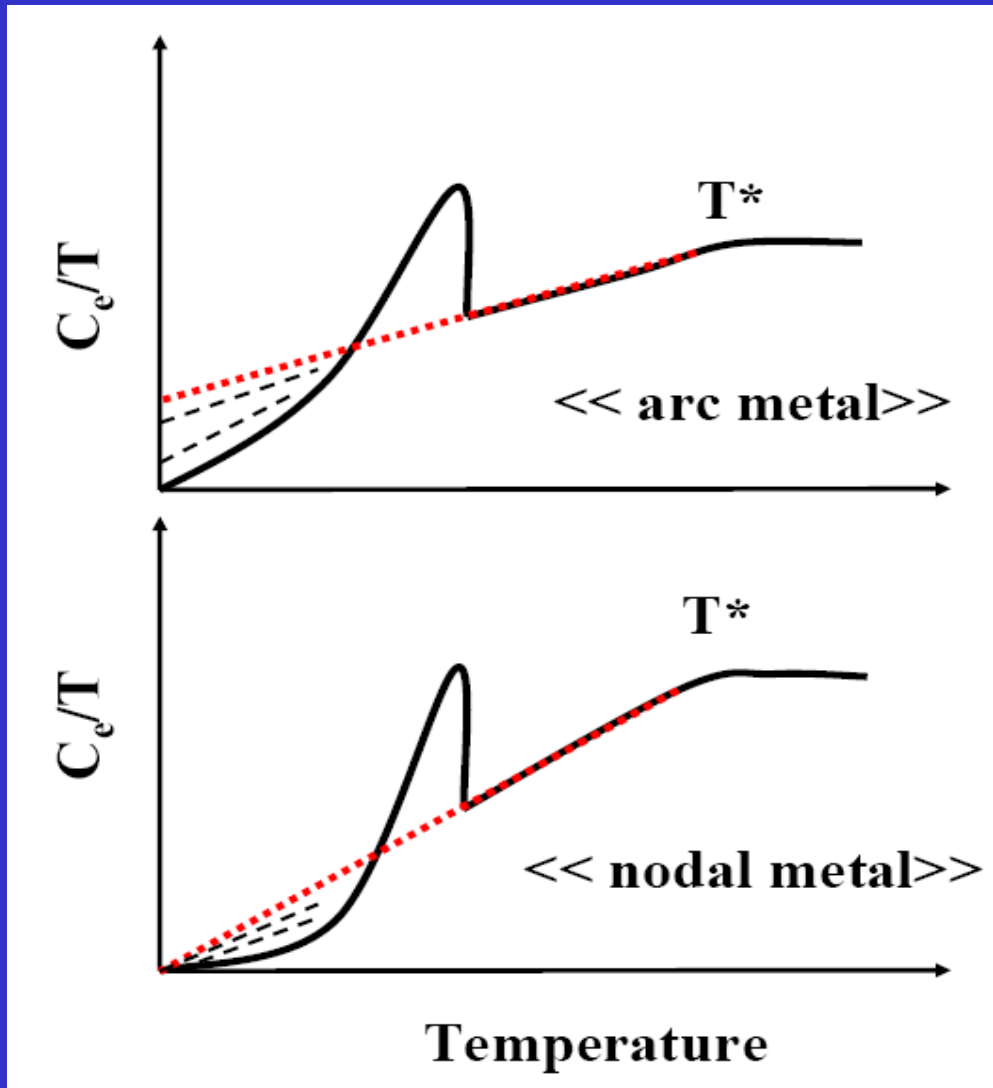


T = 15 K

K. M. Shen, et al., Science 307, 901(2005).



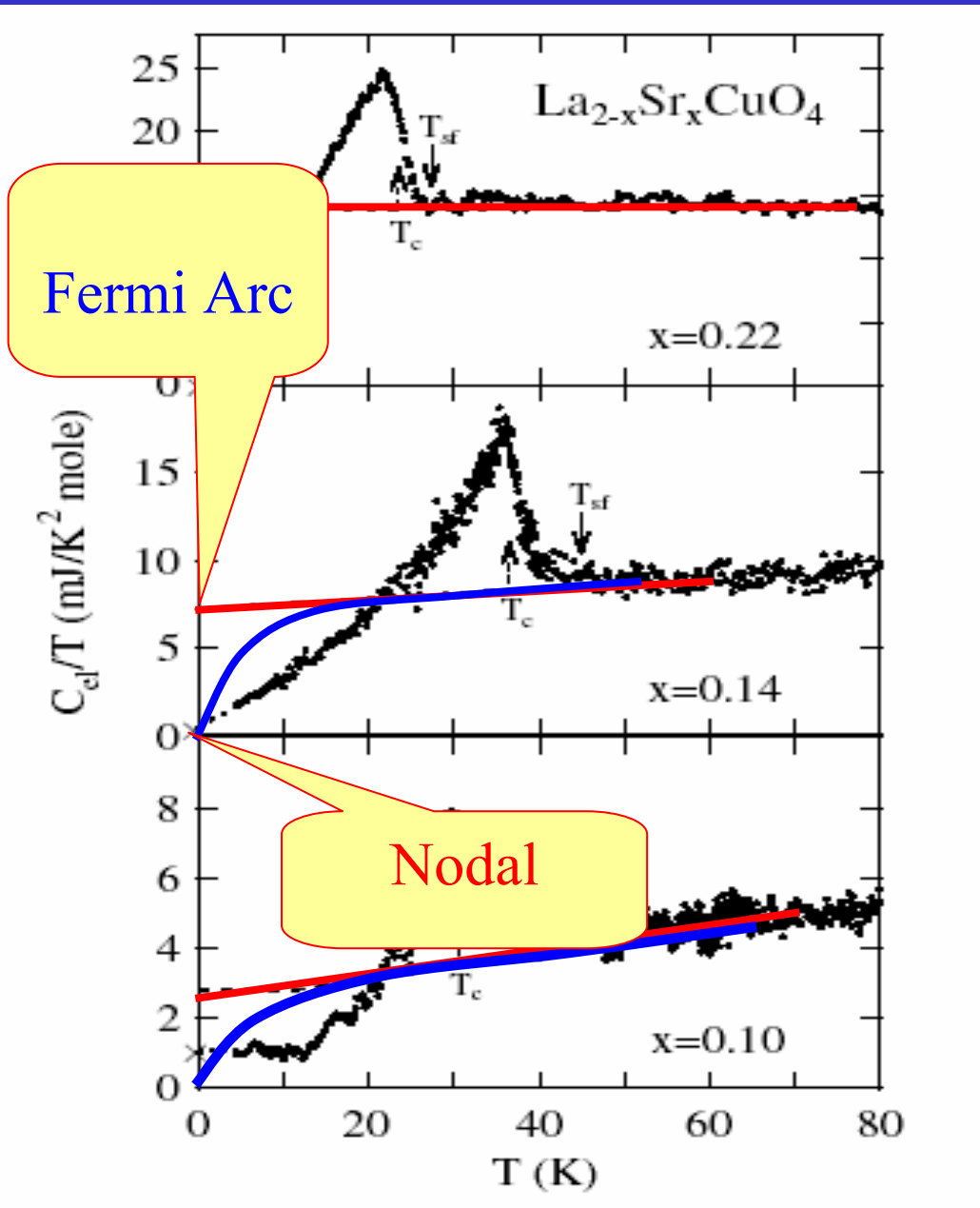
There should be Fermi arcs in the pseudogap state if the superconductivity would be suppressed completely!



$$C_e/T \approx a(\gamma_0 + bT)$$

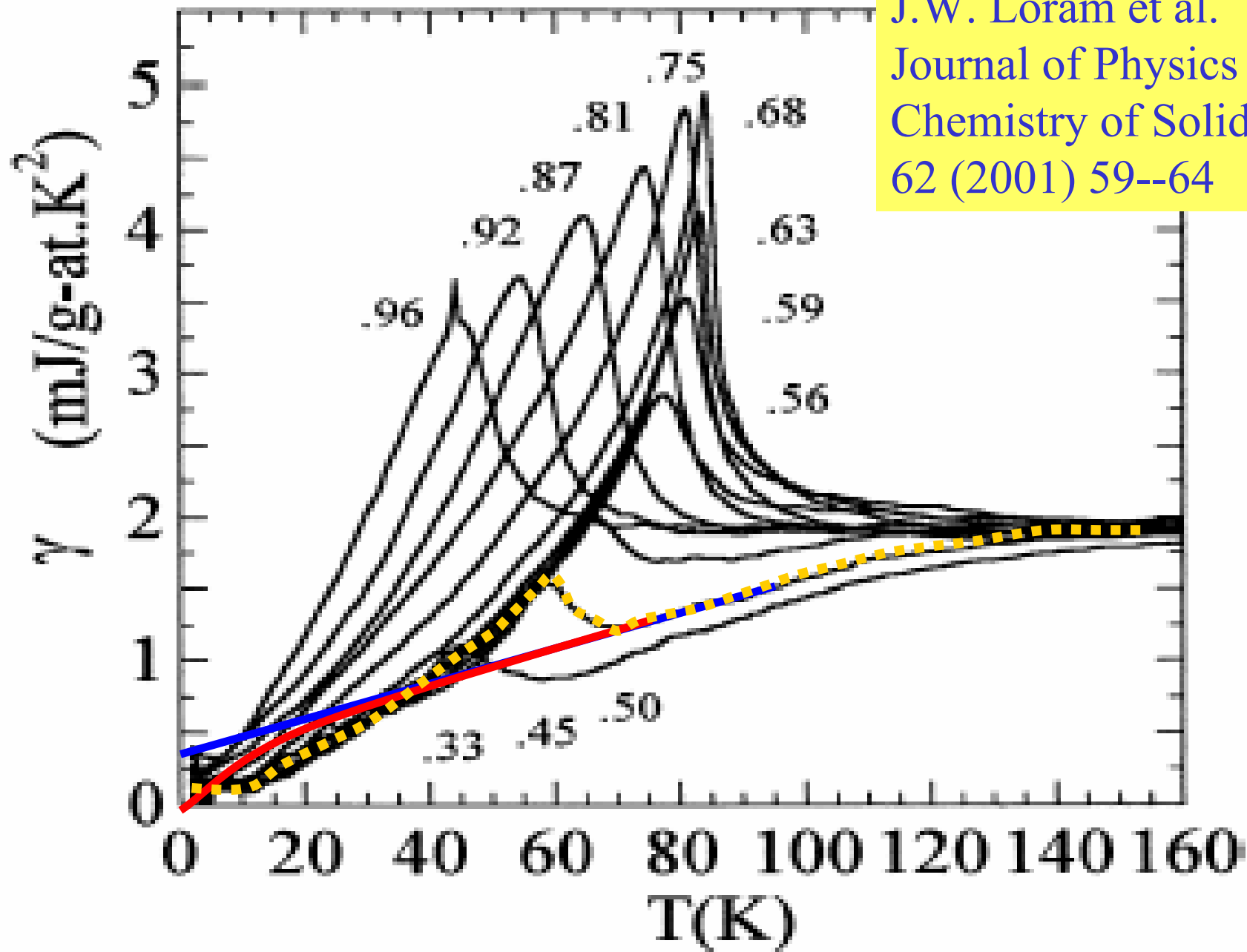
$$N(E) = N(0) \int \frac{d\theta}{2\pi} \operatorname{Re} \frac{E}{\sqrt{E^2 - \Delta_0^2 \cos^2(2\theta)}}$$

$$C_e/T = a \frac{\gamma_n T}{T_c}$$



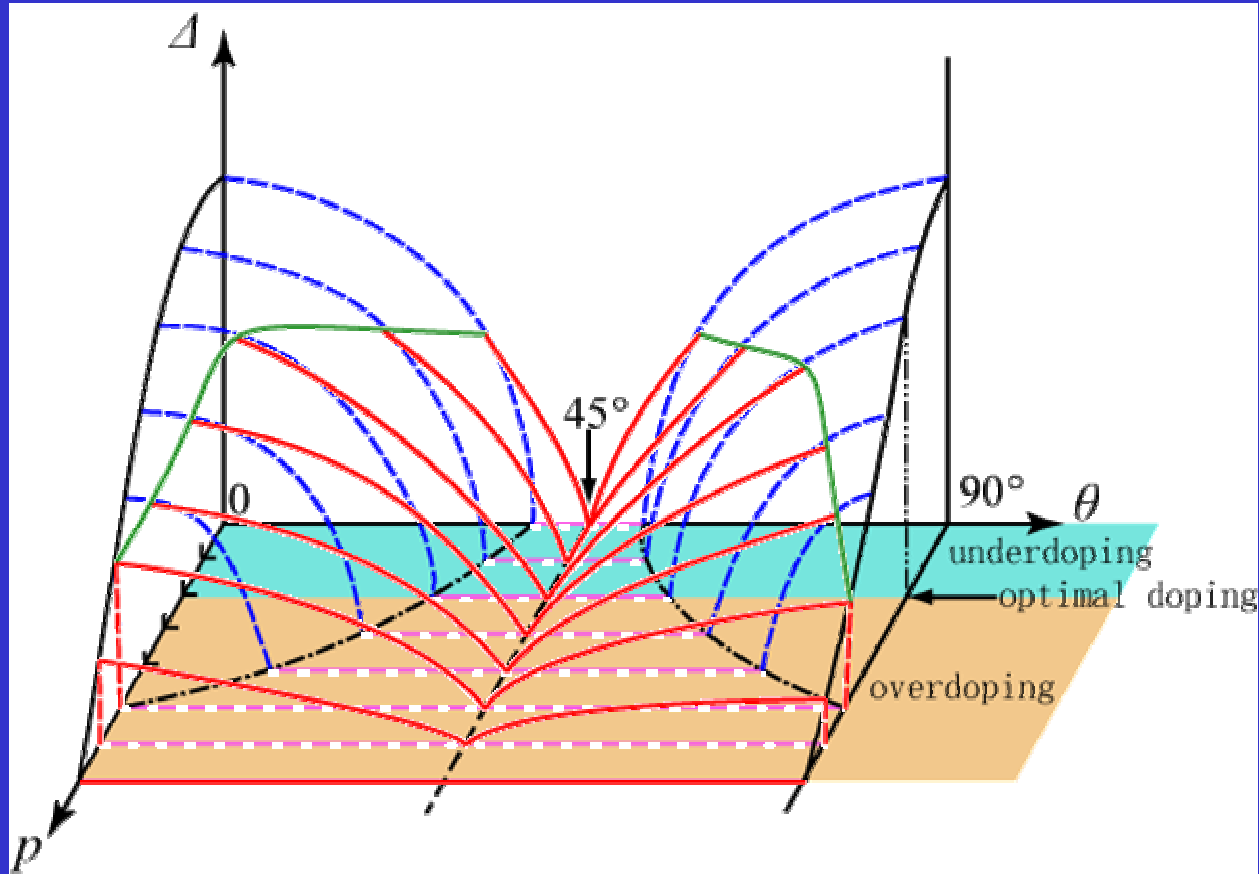
T. Matsuzaki, N. Momono, M. Oda, M. Ido, J. Phys. Soc. Japn. 73, 2232(2004).

From the entropy point of view, the nodal metal picture for the PG phase is not reasonable.



J.W. Loram et al.
Journal of Physics and
Chemistry of Solids
62 (2001) 59--64

What governs the
superconducting
transition temperature?

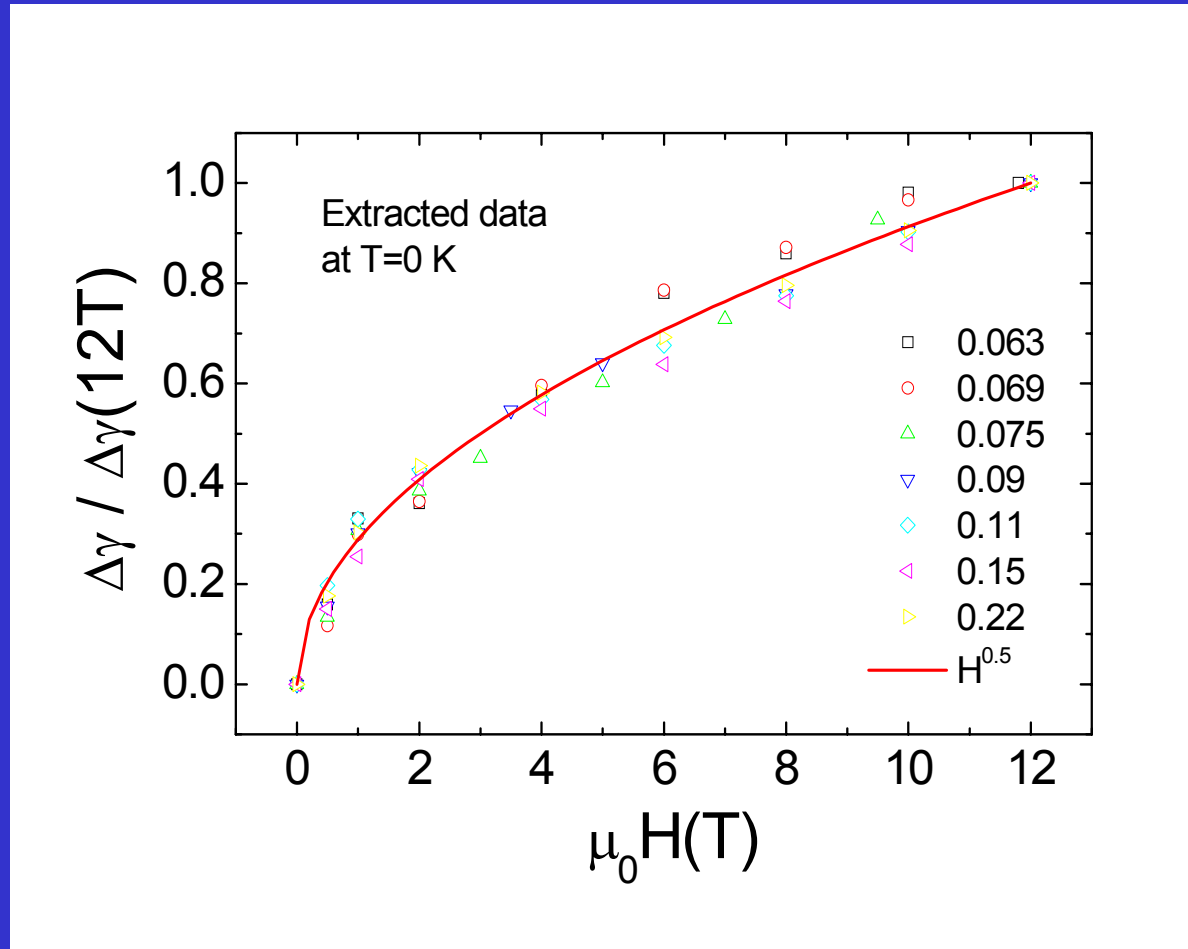


$$\Delta_s = \frac{\frac{1}{2} k_{arc}}{k_F} \left[\frac{d\Delta_s}{d\phi} \right]_{node}$$

$$= \frac{1}{2} v_{\Delta} \hbar k_{arc} \approx k_B T_c$$

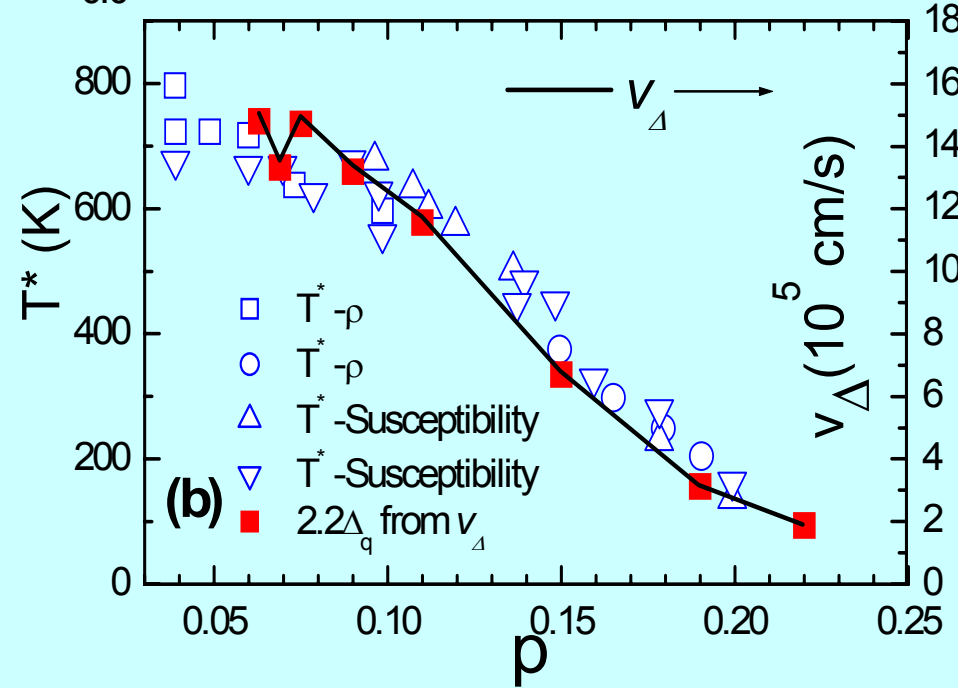
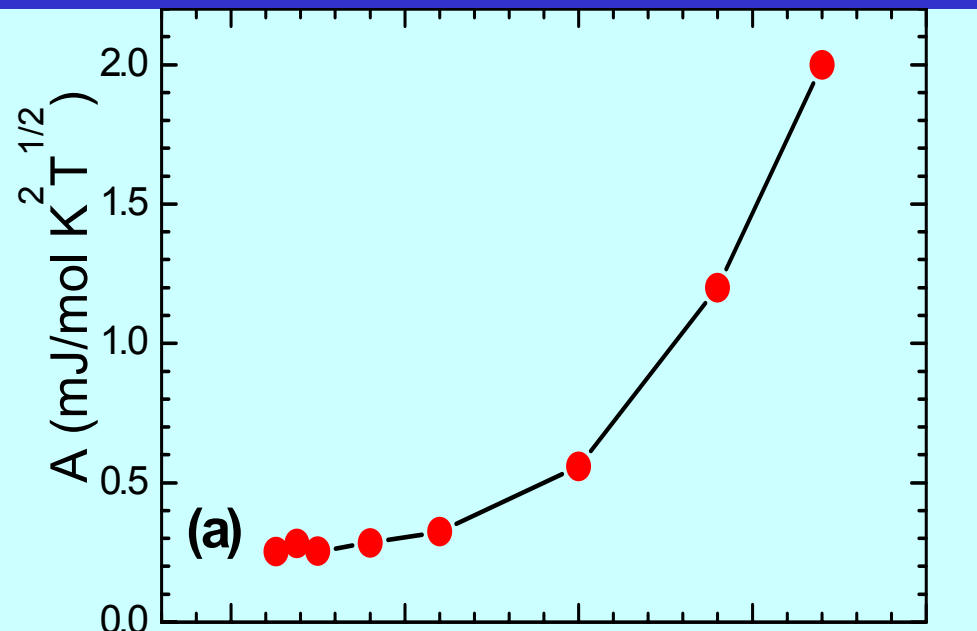
Good scaling to Volovik's relation at $T=0$ for all doping concentrations:

$$\gamma_{vol} = A\sqrt{H}$$



H. H. Wen, et al., Phys. Rev. B70, 214505(2004).

Four Nodal points:



$$\gamma_{vol} = A\sqrt{H}$$

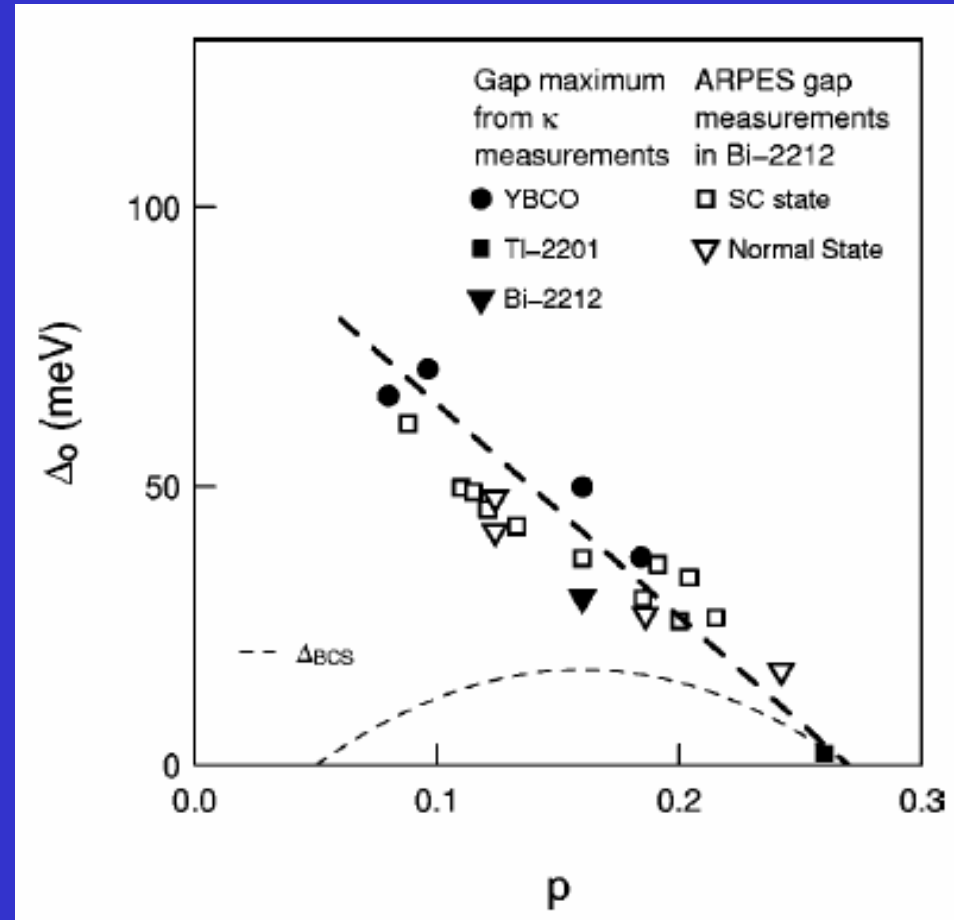
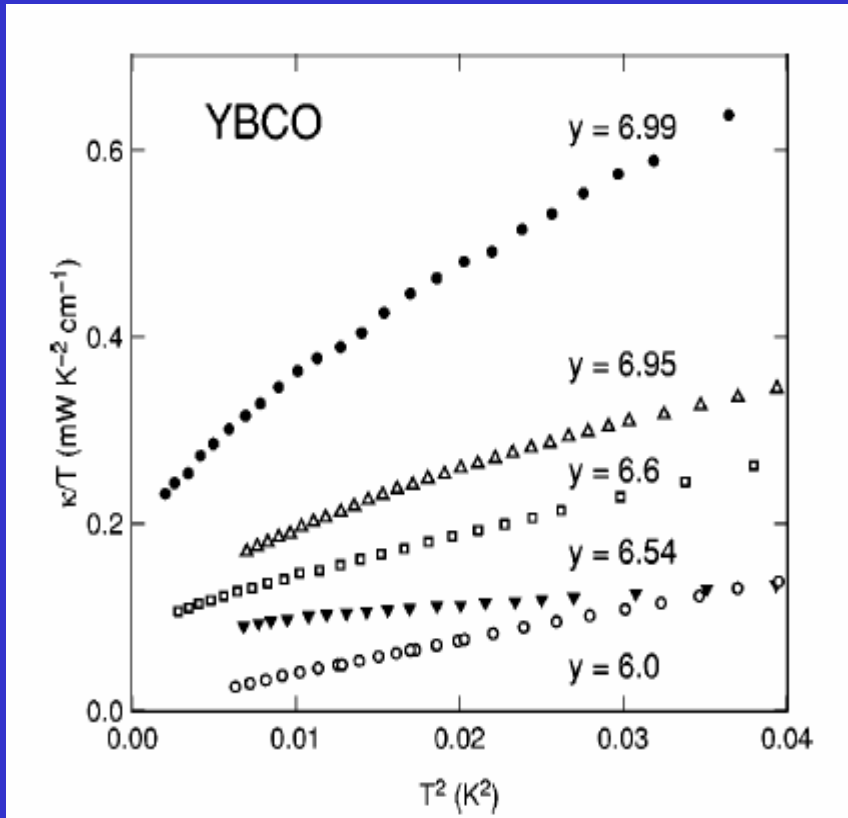
$$A = \alpha_p \frac{4k_B^2}{3\hbar l_c} \sqrt{\frac{\pi}{\Phi_0}} \frac{nV_{mol}}{v_\Delta}$$

$$v_\Delta = \left[\frac{d\Delta_s}{d\phi} \right]_{node} / \hbar k_F$$

$$\Delta_q = v_\Delta \hbar k_F / 2$$

$$2.2\Delta_q = k_B T^*$$

Open symbols: T. Timusk, B. Statt, Rep. Prog. Phys. 62, 61(1999).

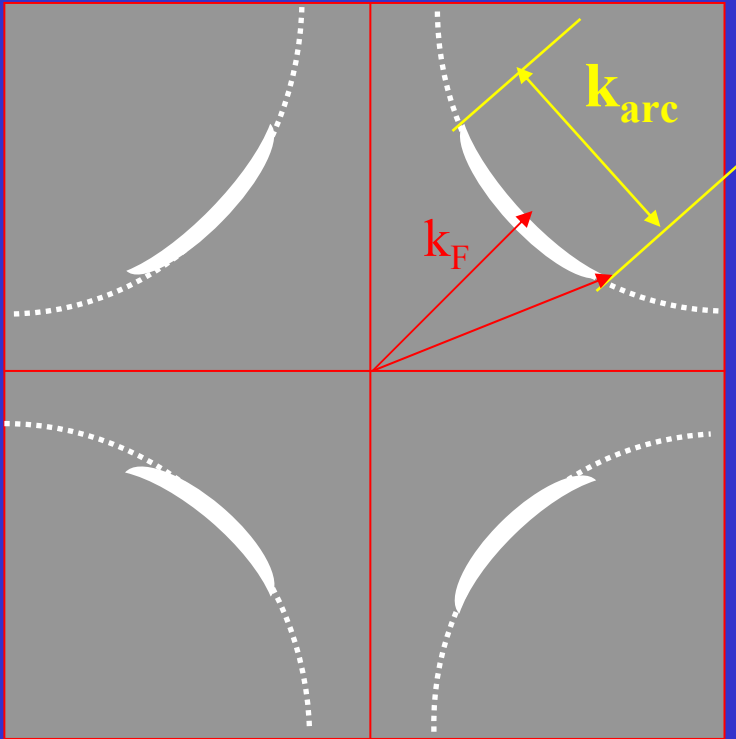


$$\frac{\kappa_0}{T} = \frac{k_B^2}{3\hbar} \frac{n}{d} \left(\frac{v_F}{v_\Delta} + \frac{v_\Delta}{v_F} \right)$$

M. Sutherland...L. Taillefer, Phys. Rev. B67, 174520(2003)

Superconducting Energy Scale:

$$\Delta_s = \frac{1}{2} v_\Delta \hbar k_{arc}$$



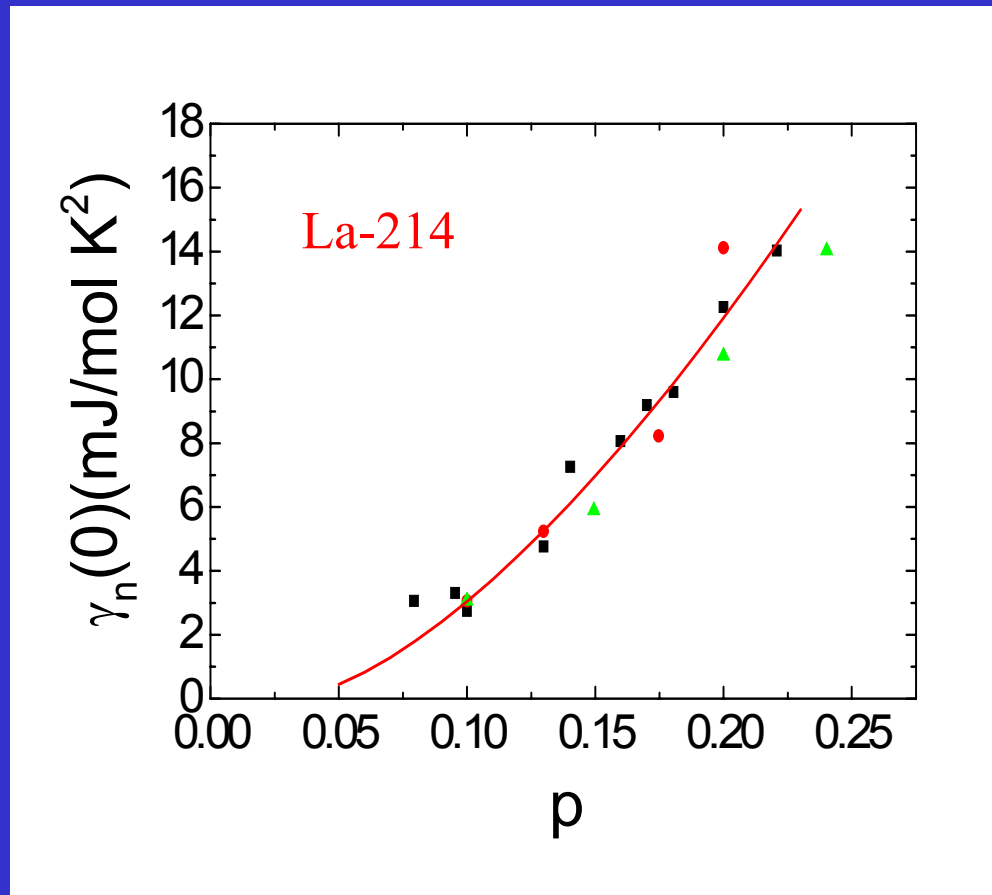
For a 2D Fermi surface

$$\gamma_n(0) = 2nk_B^2 k_{arc} V_{mol} / 3\hbar v_F l_c$$

$$T_c = \alpha_s^{-1} \frac{3\hbar^2 v_F l_c \gamma_n(0) v_\Delta}{4nk_B^3 V_{mol}} = \beta \gamma_n(0) v_\Delta$$

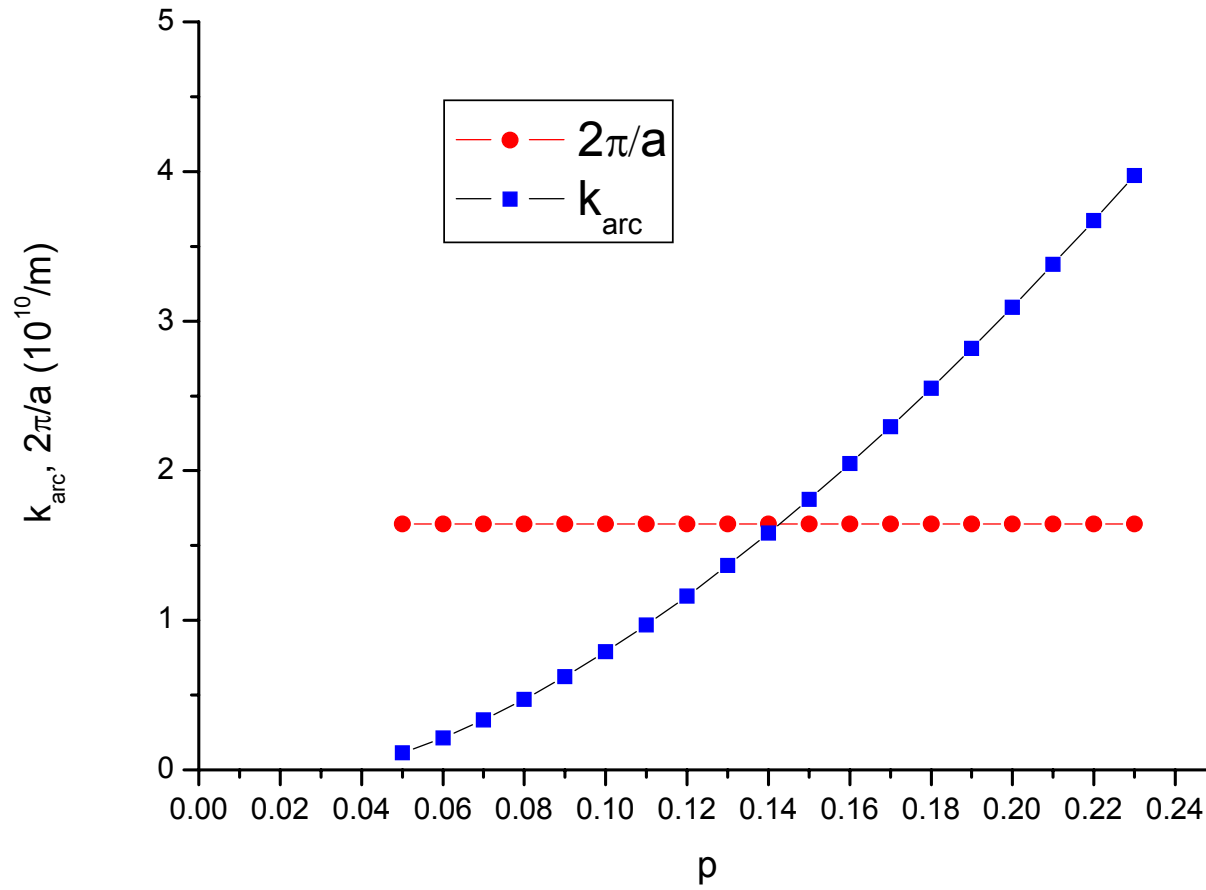
The residual DOS at zero temperatures

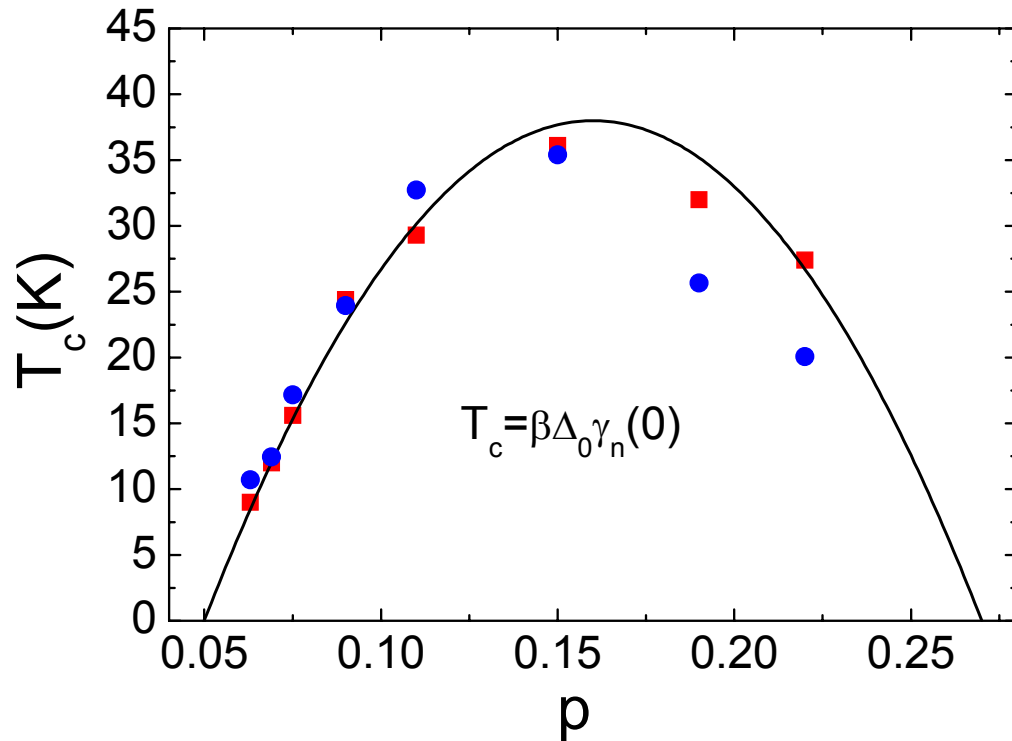
$$\gamma_n(0) = 182.6(p - 0.03)^{1.54}$$



T. Matsuzaki, N. Momono, M. Oda, M. Ido, J. Phys. Soc. Japn. 73, 2232(2004).

Comparison between k_{arc} and $2\pi/a$





$$\beta = 0.7445 K^3 \text{ mols} / Jm$$

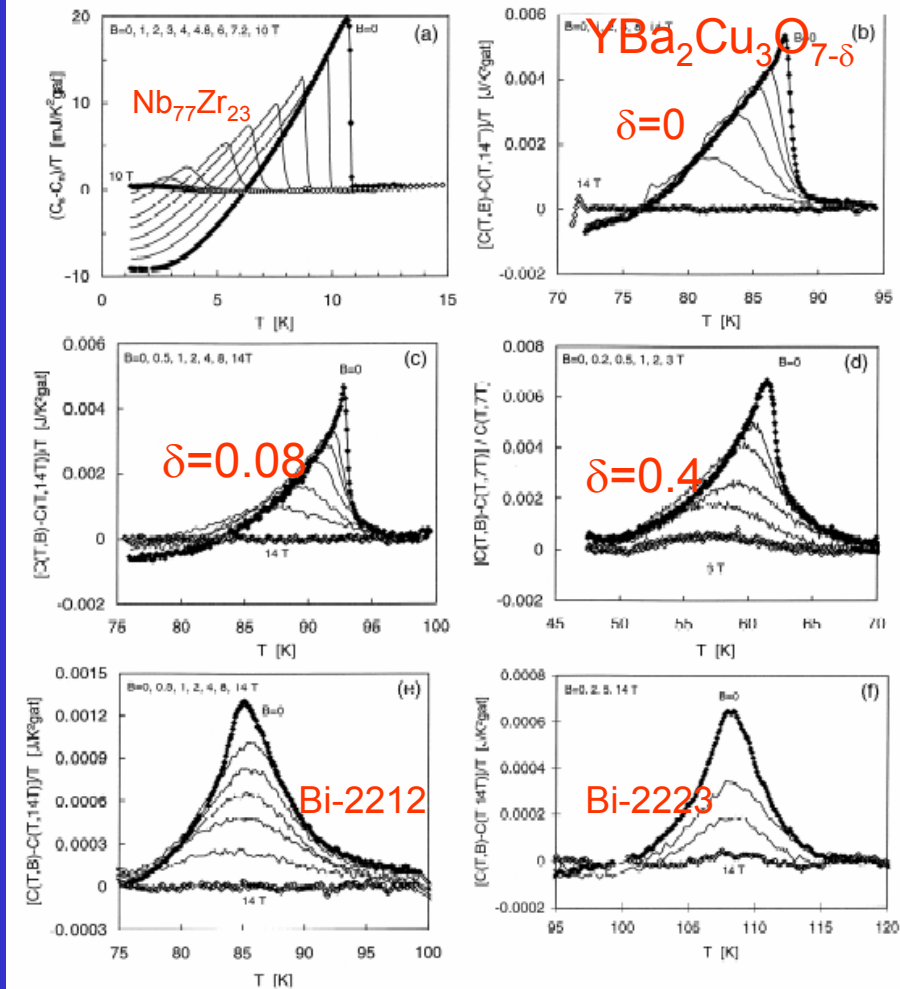
$$v_F = 2.73 \times 10^7 \text{ cm} / s$$

$$\alpha_s = 13.8$$

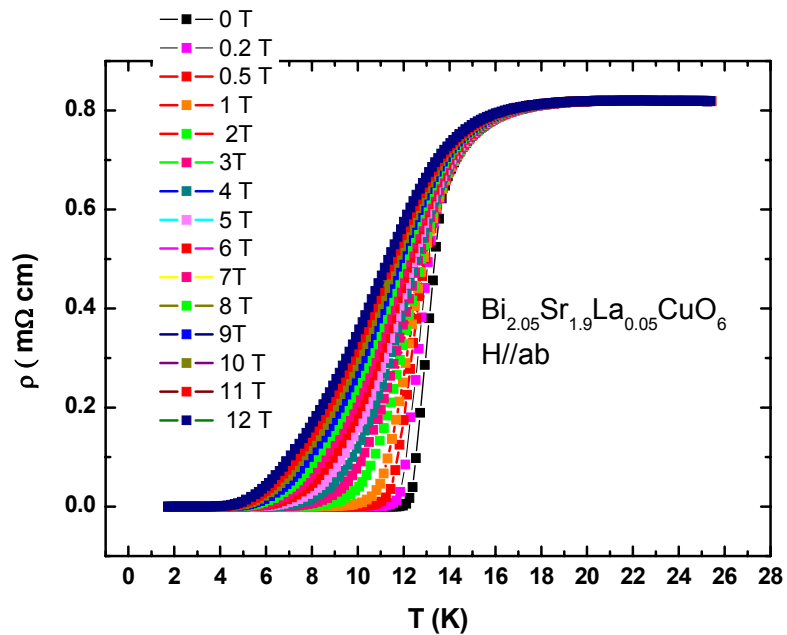
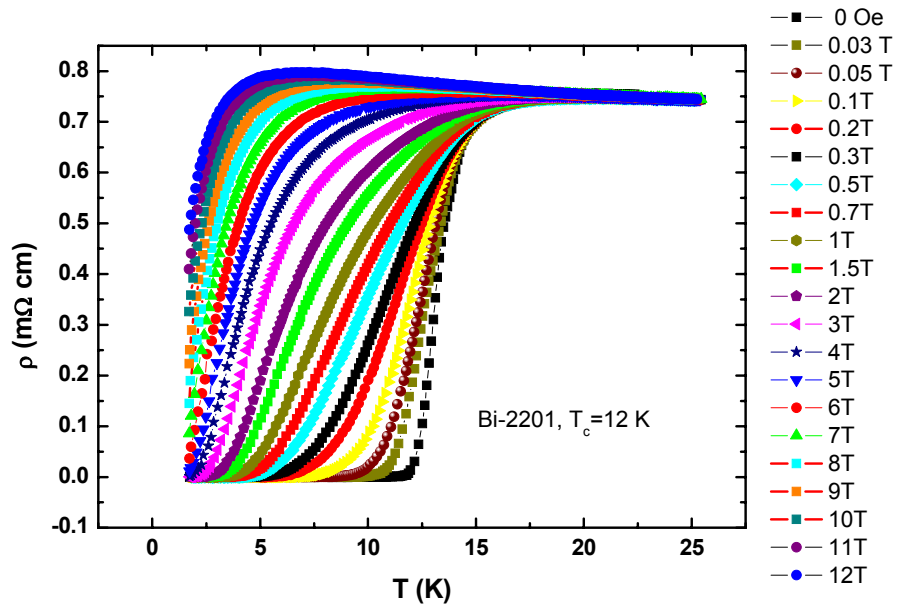
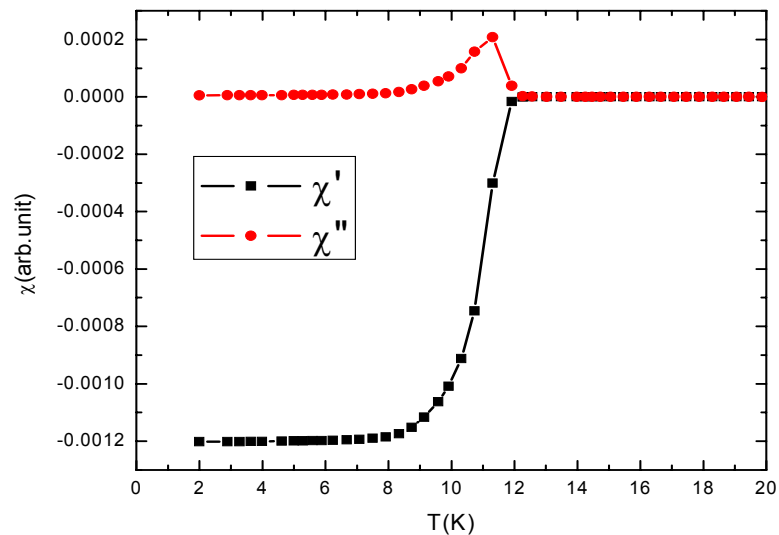
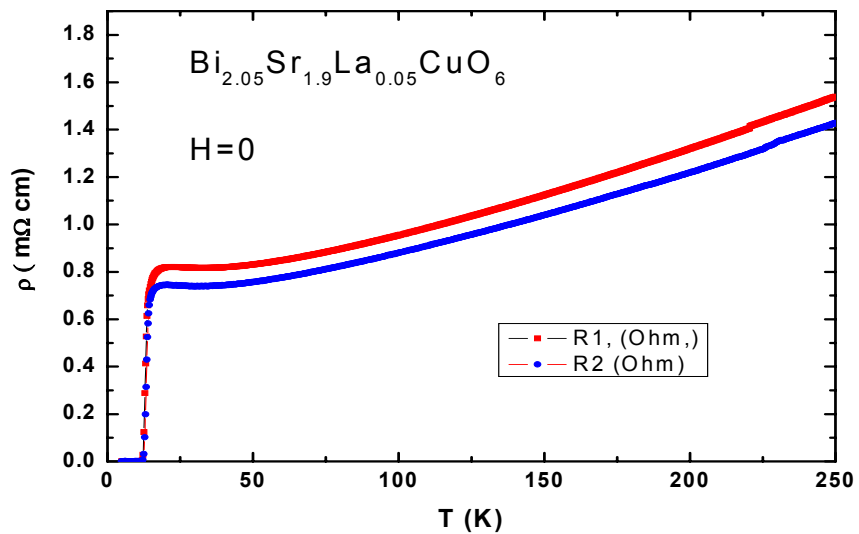
$$T_c = \alpha_s^{-1} \frac{3\hbar^2 v_F l_c \gamma_n(0) v_\Delta}{4nk_B^3 V_{mol}} = \beta \gamma_n(0) v_\Delta$$

H. H. Wen, et al., Phys. Rev. B 72, 134507(2005).

Non BCS behavior in underdoped regime

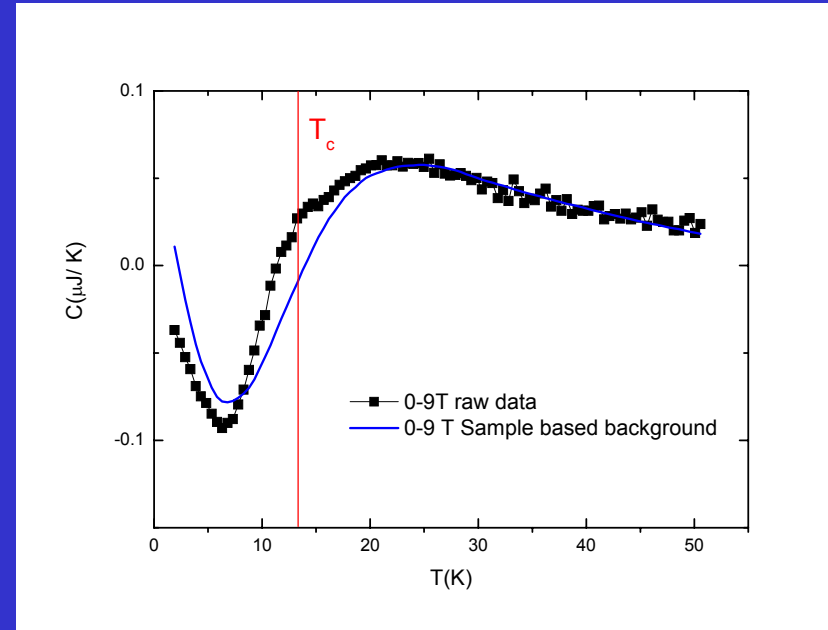
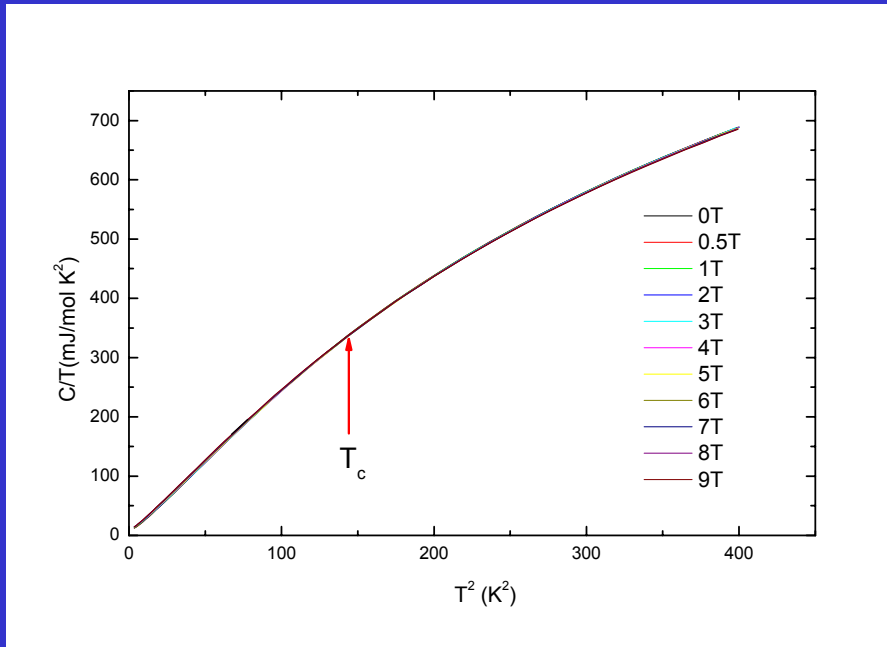


A. Junod, A. Erb, and C. Renner,
 Physica C 317-318, 333 (1999).

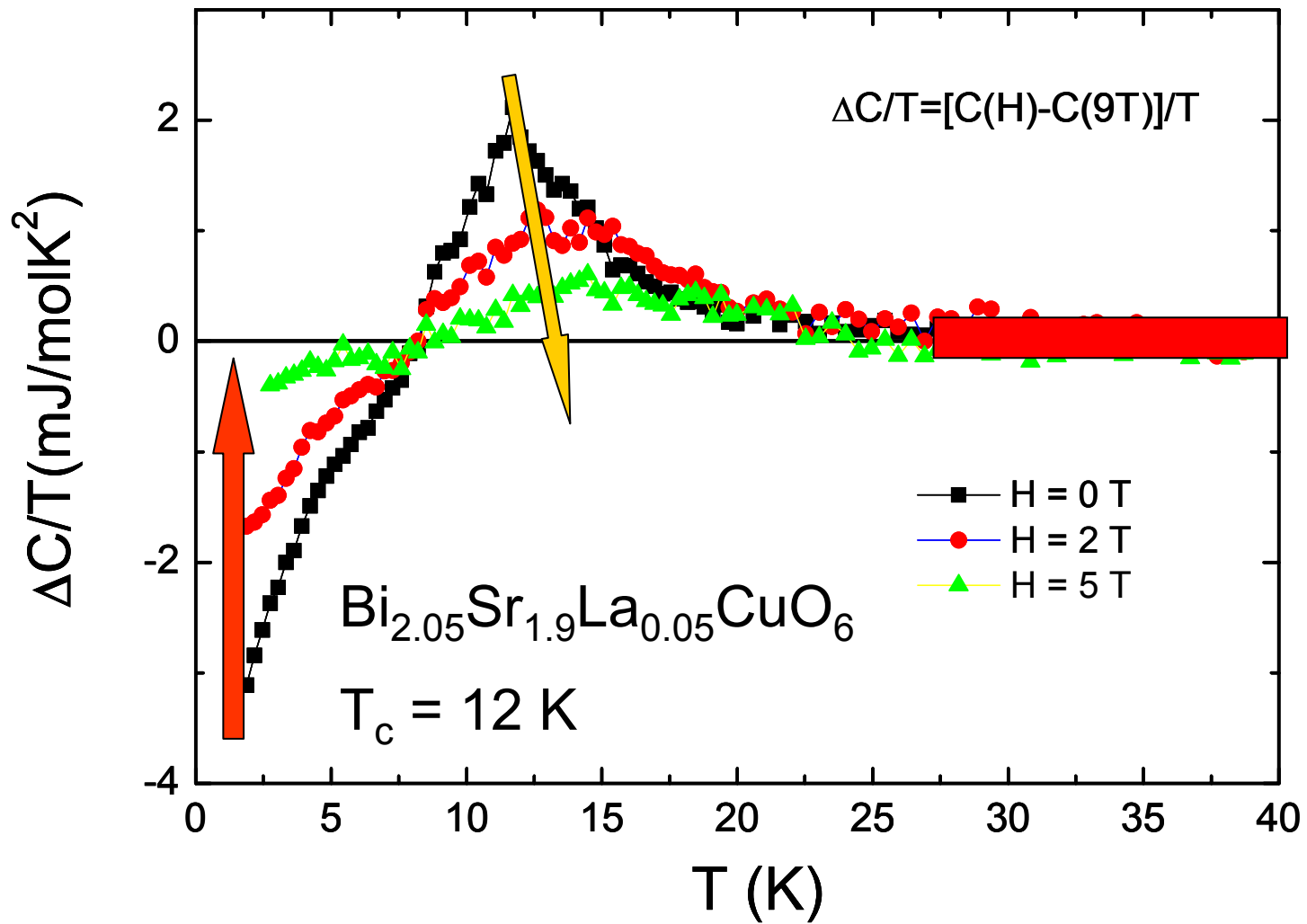


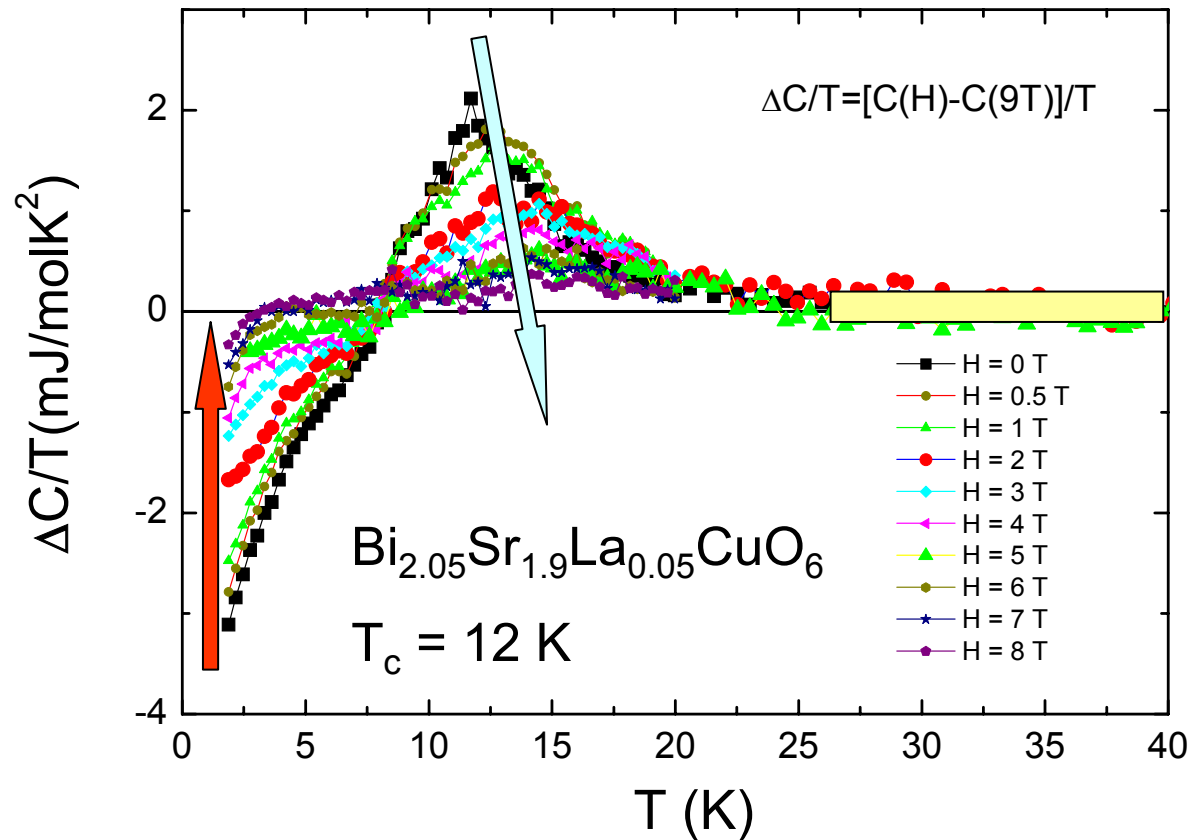
To get useful message from “nothing”

Improved Low-temperature specific heat measurements based on PPMS

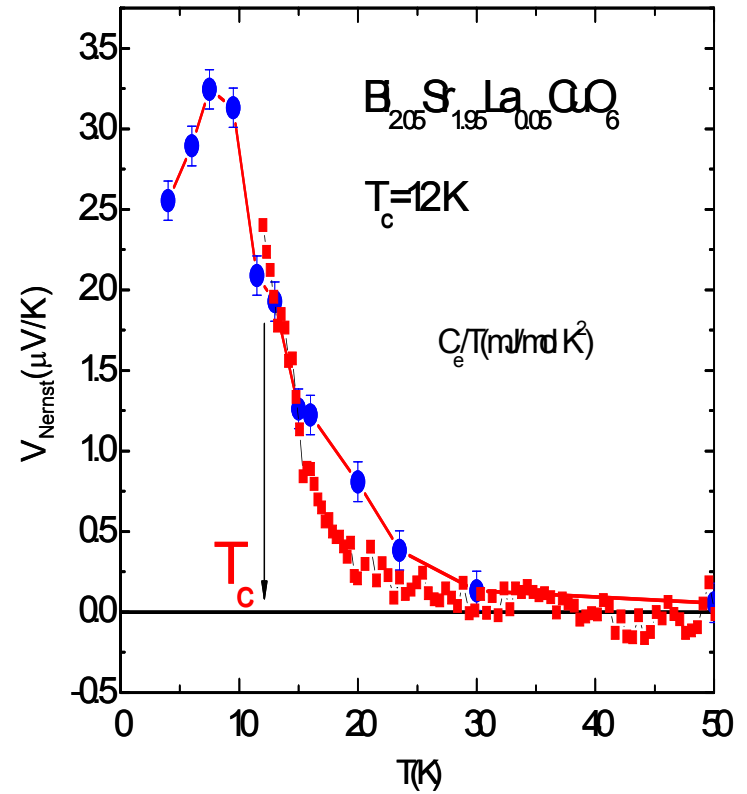
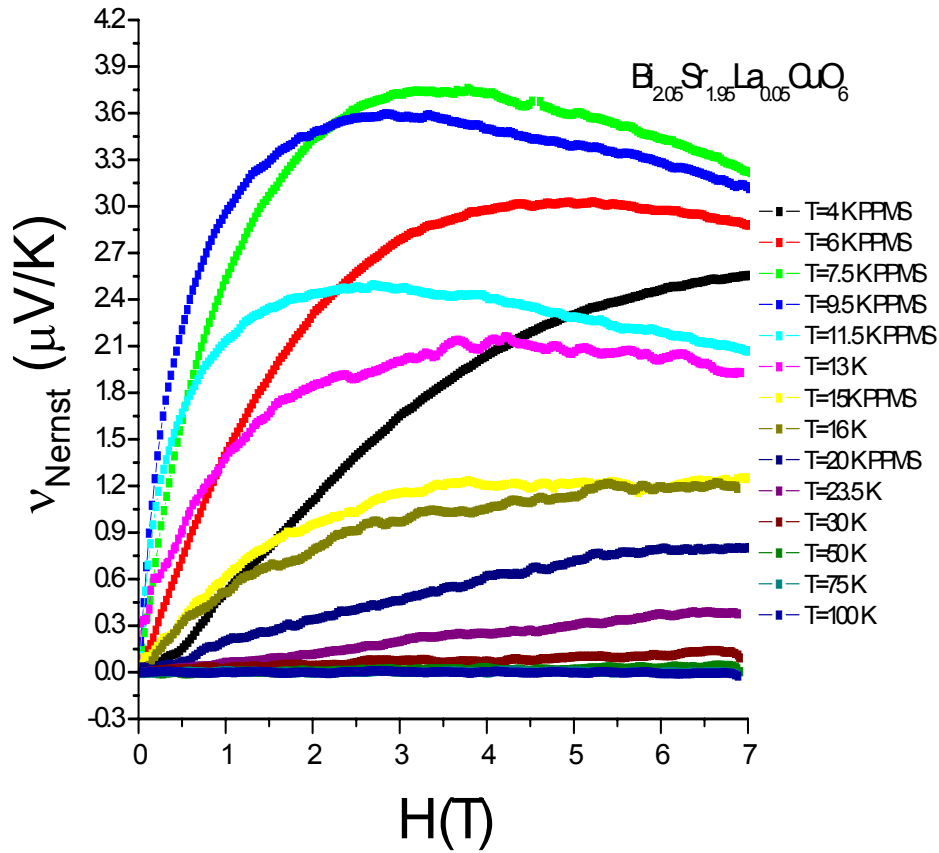


After a complicated process, (inc. data acquisition and treatment), we finally get the useful message!



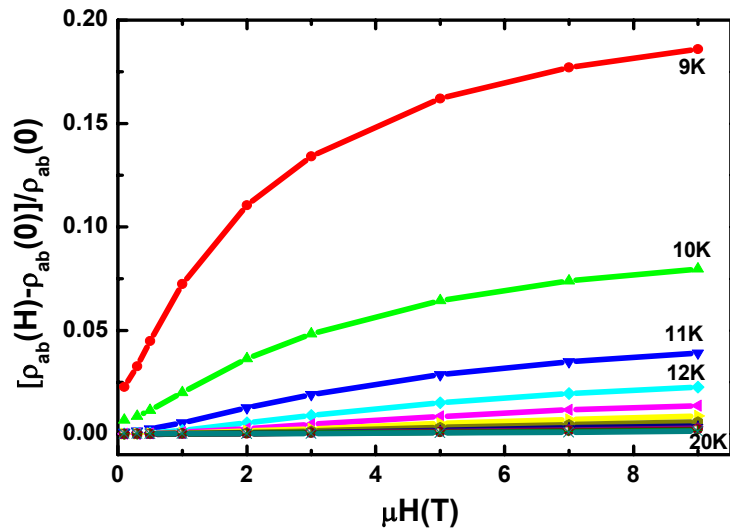
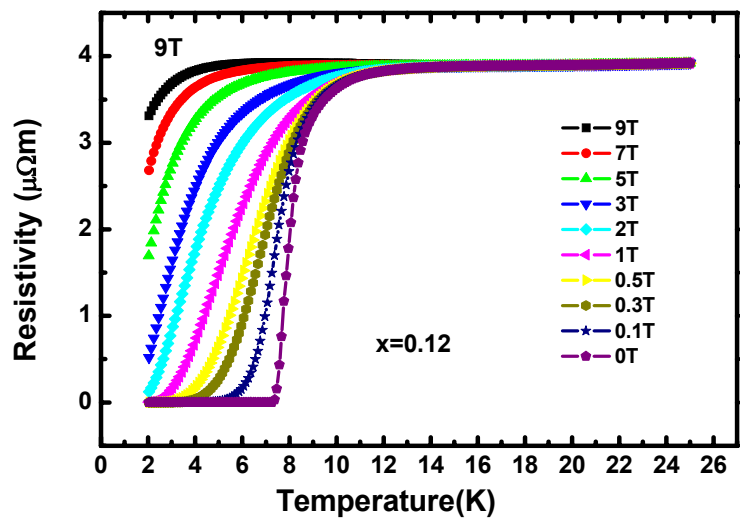
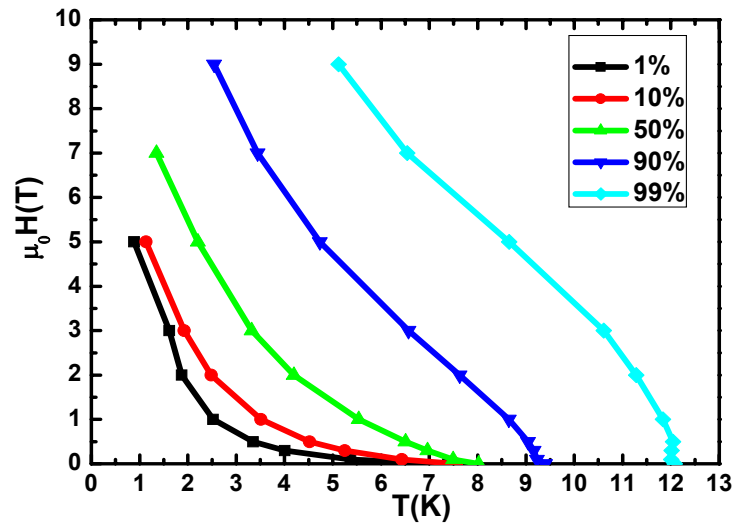
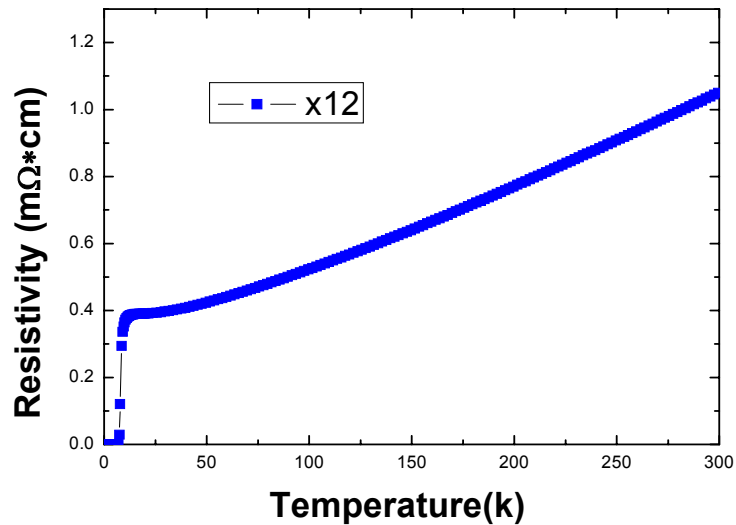


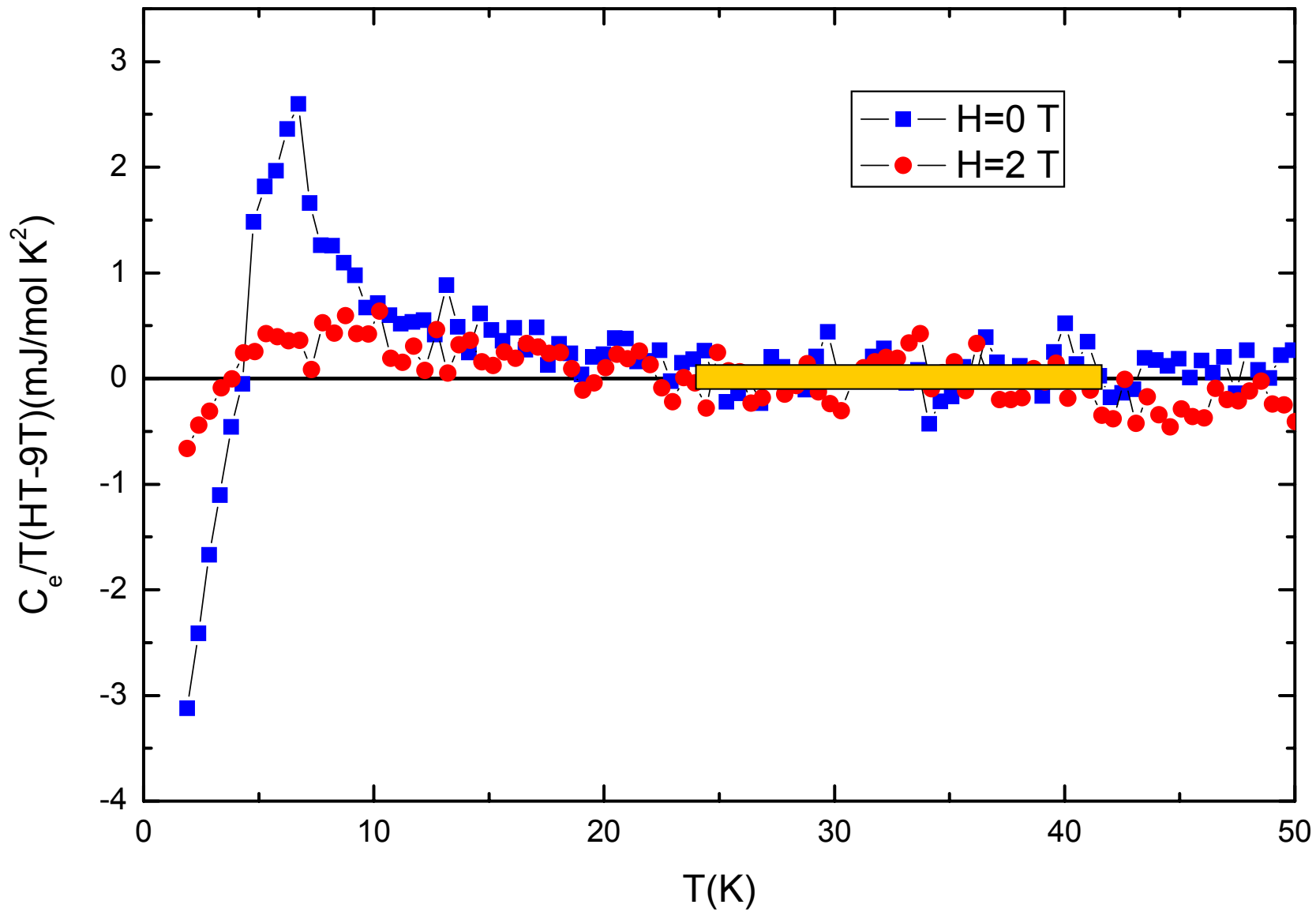
- Cooper pairs are formed far above T_c : Gaussian fluctuation? Or BEC or Phase fluctuation, or something else?
- At $T=0 \text{ K}$, field will induce finite DOS, a Fermi arc (or pocket) as the ground state of pseudogap state?

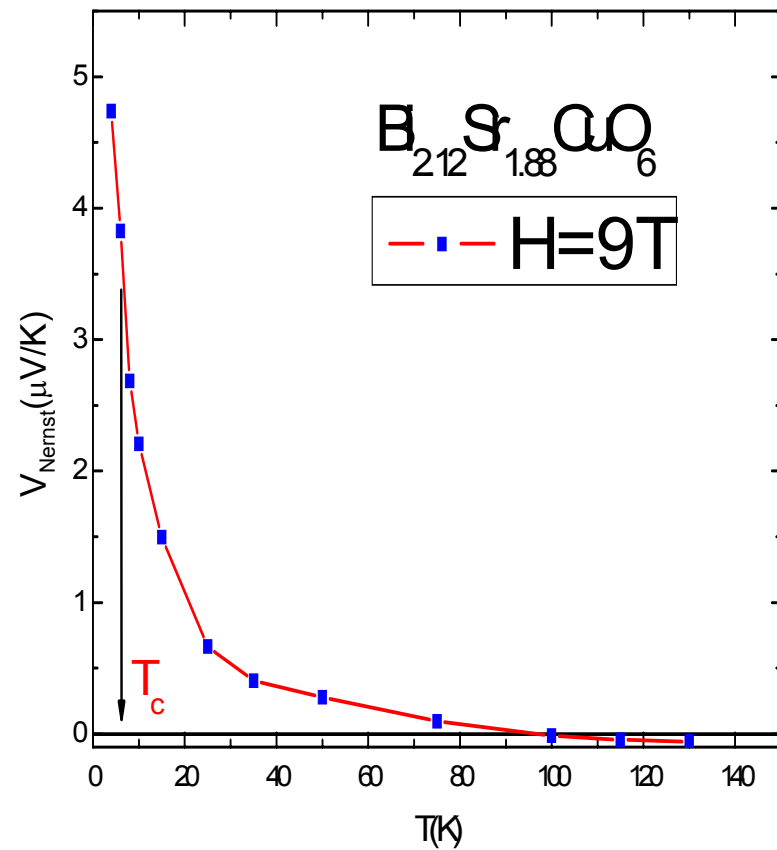
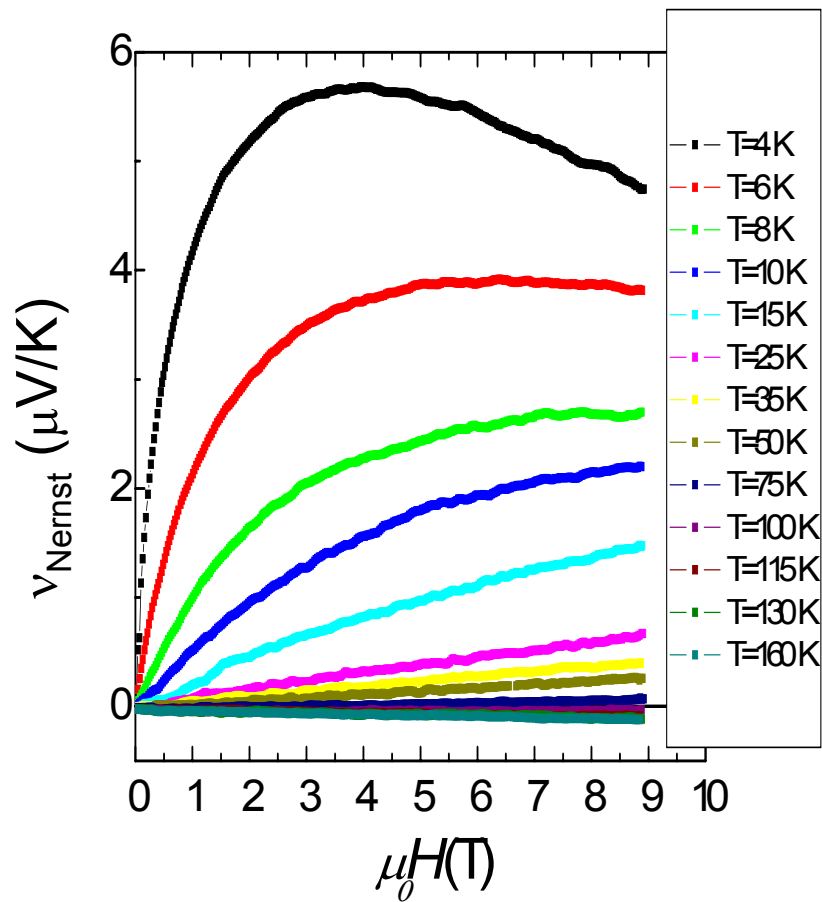


Fluctuating superconductivity remains up to about 30-40 K, far above $T_c = 12\text{K}$

$\text{Bi}_{2+x}\text{Sr}_{2-x}\text{CuO}_6$ single crystals

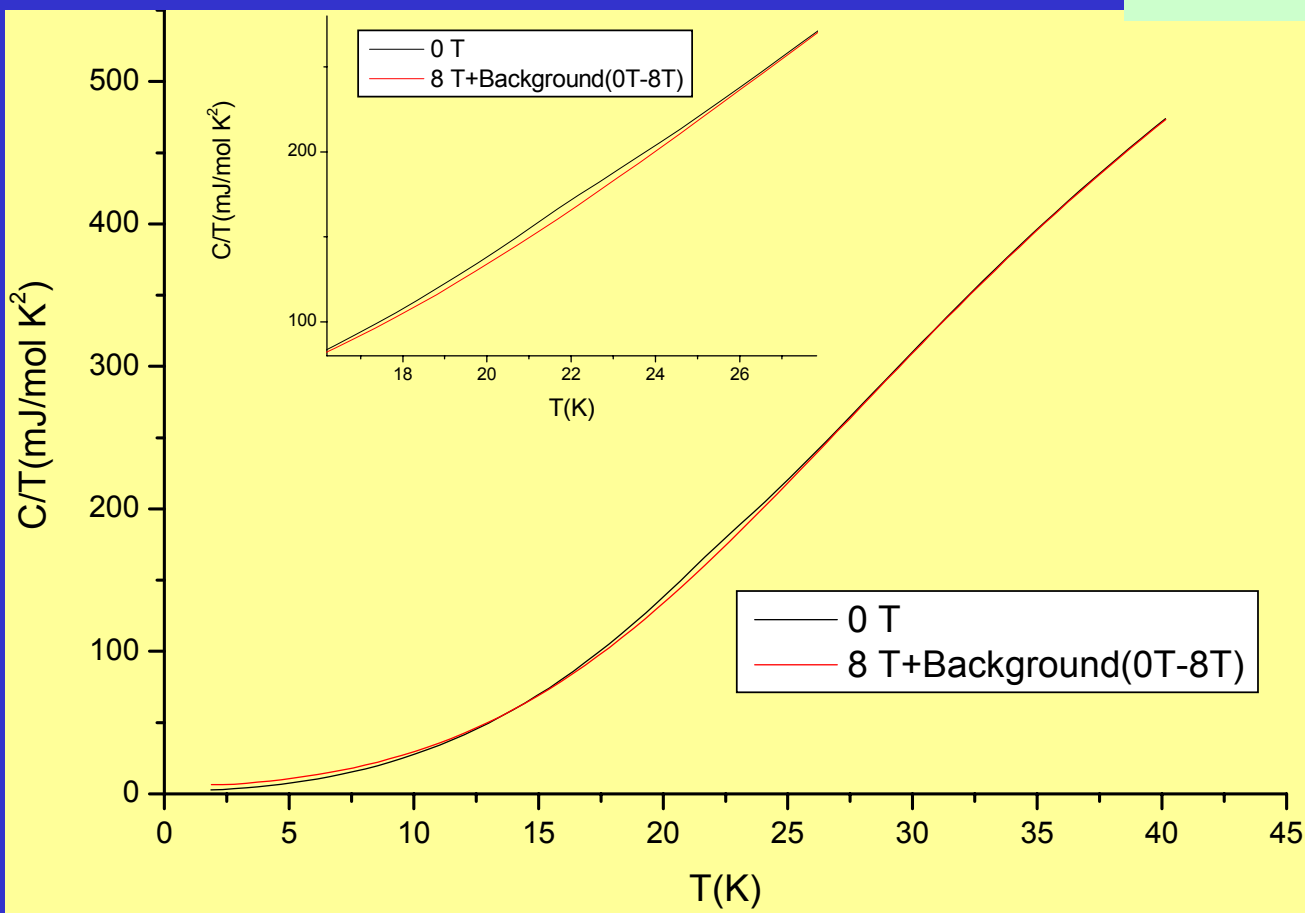
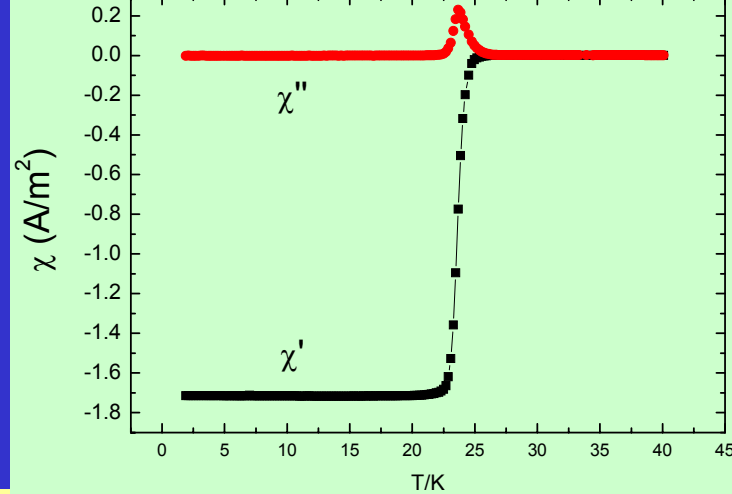


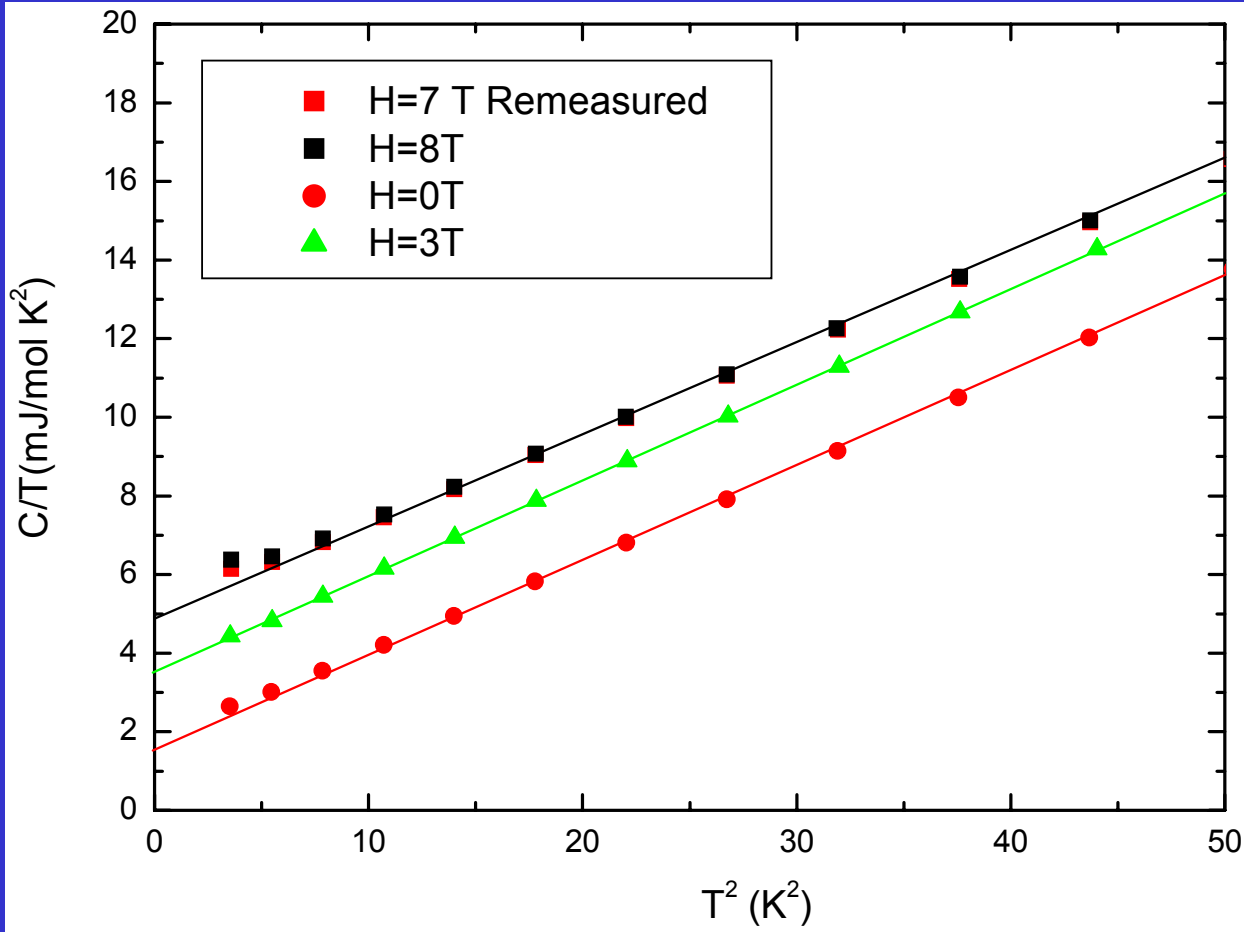




Electron Doped Region

PrLaCeCuO, $T_c=25$ K





$$\frac{C}{T} = \gamma_n + \beta T^2$$

$$\gamma_n = 4.88 \text{ mJ/mol K}^2$$

$\gamma_0 = 1.60 \text{ mJ/mol K}^2$ (perhaps due to the impurity scattering? Nodal gap?)

$$\beta = 0.2337 \text{ mJ/mol K}^4$$

Phonon Mediated Pairing?

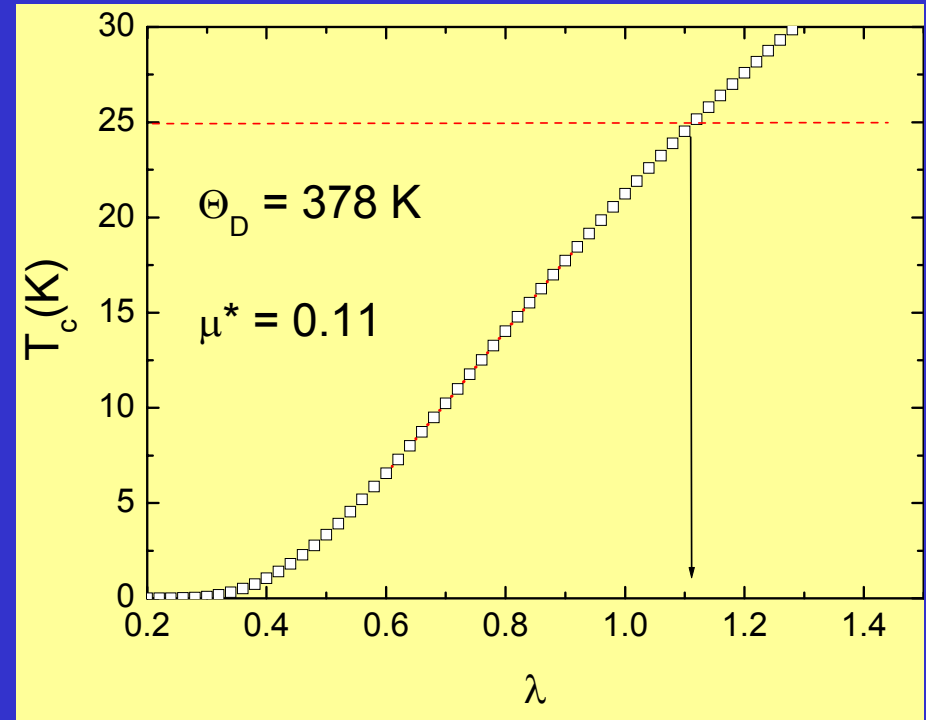
$$\Theta_D = (12\pi^4 k_B N_A Z / 5\beta)^{1/3}$$

$$\beta = 0.233 \text{ mJ} / \text{molK}^4$$

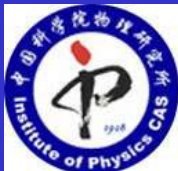
$$Z = 7$$

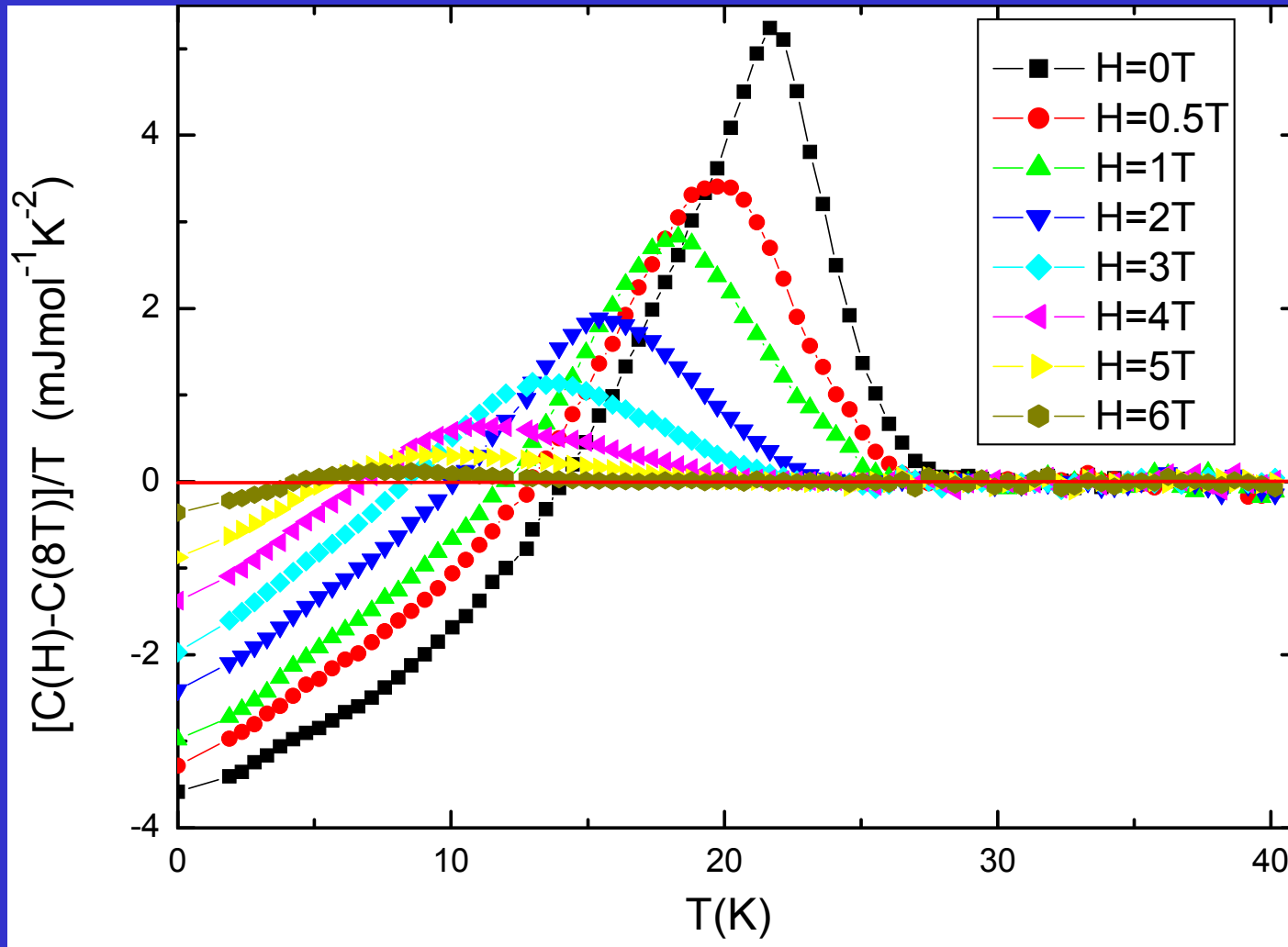
$$\Theta_D = 387 \text{ K}$$

$$T_c = \frac{\Theta_D}{1.45} \exp\left[-\frac{1.04(1+\lambda)}{\lambda - \mu^*(1+0.62\lambda)} \right]$$



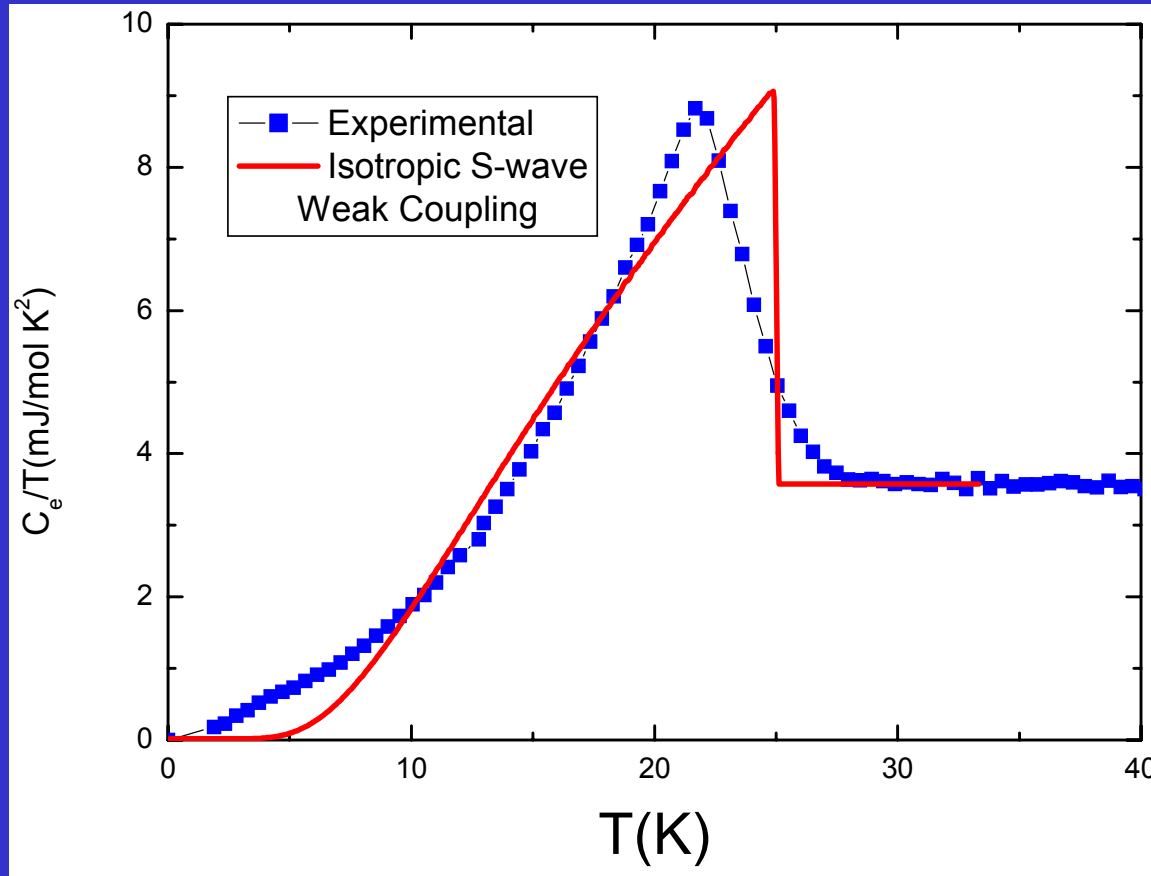
The Debye temperature does not change with the doping. It seems that the MacMillan equation based on e-ph cannot explain the T_c in PLCCO





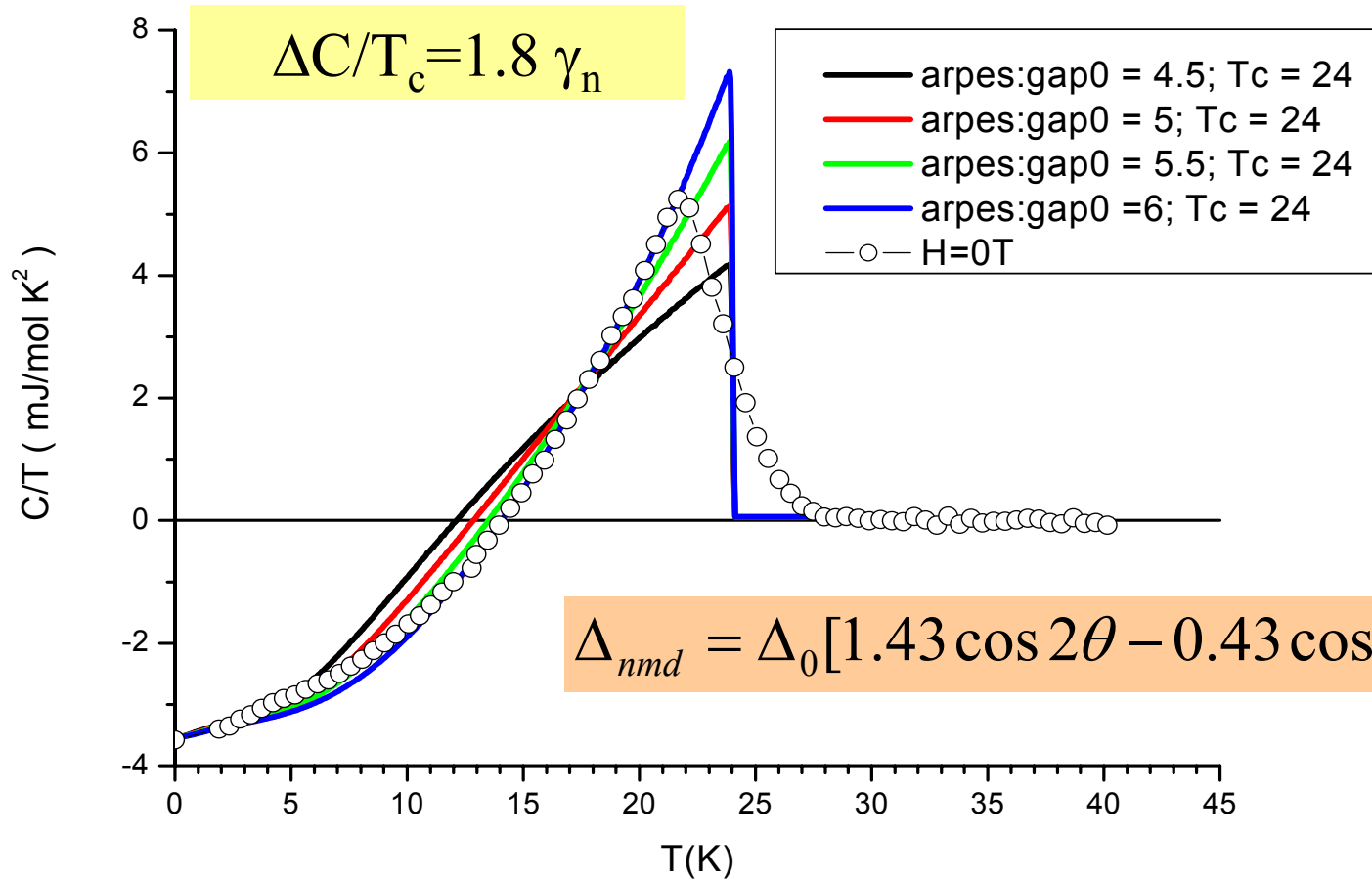
The low temperature part shows a power law, not an exponential dependence! Indicating either anisotropic or d-wave gap symmetry.

Failure to an isotropic s-wave gap



$$\gamma_e = \frac{4N(E_F) \hbar\omega_D}{k_B T^3} \int_0^{\hbar\omega_D} d\varepsilon \left[\varepsilon^2 + \Delta^2(T) - \frac{T}{2} \frac{d\Delta^2(T)}{dT} \right] \frac{e^{\varepsilon/k_B T}}{(1 + e^{\varepsilon/k_B T})^2}$$

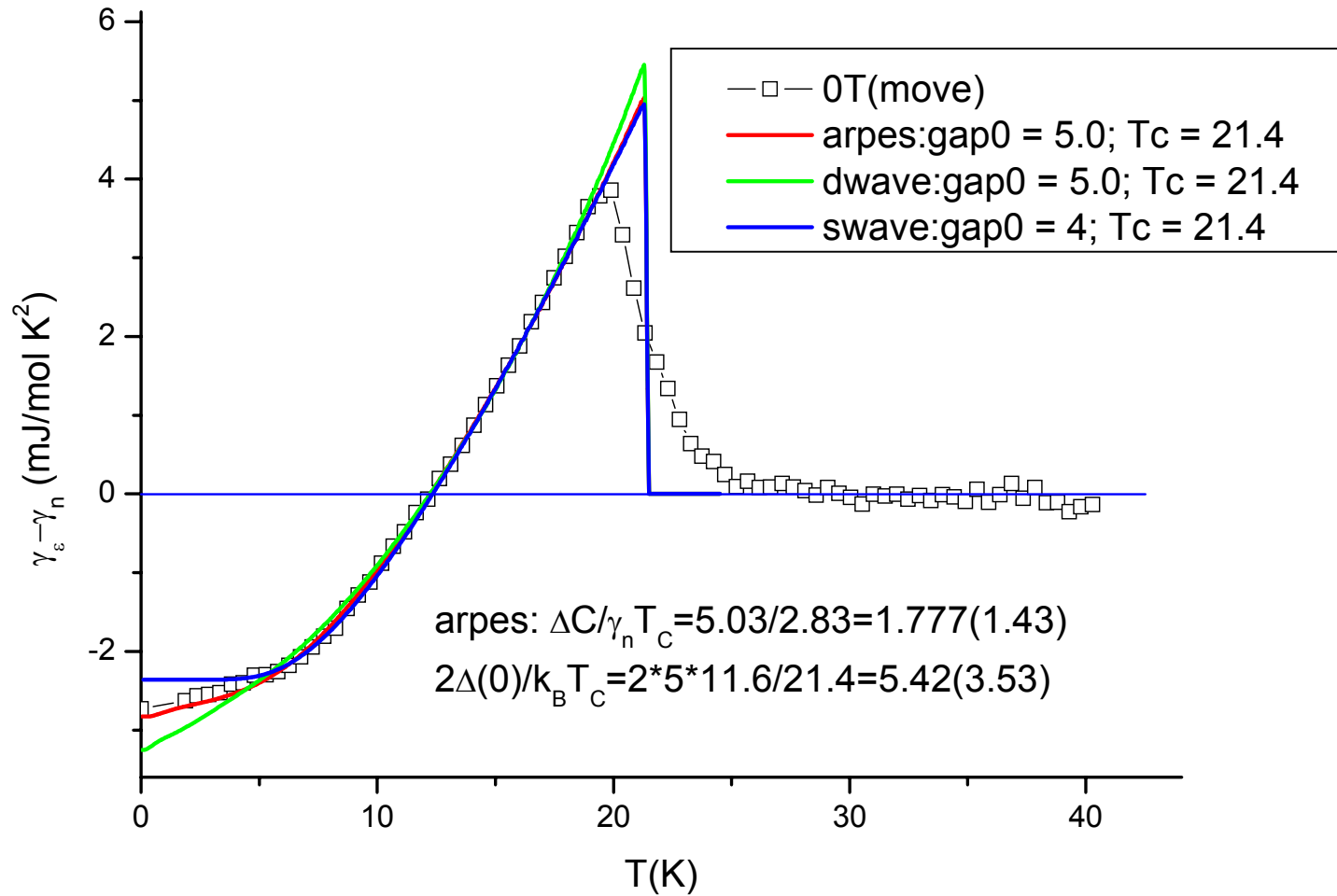
$$\zeta = \sqrt{\varepsilon^2 + \Delta^2(T)}$$



$$\gamma_e = \frac{4N(E_F)}{k_B T^3} \int_0^{\hbar\omega_D} d\varepsilon \left[\varepsilon^2 + \Delta^2(T) - \frac{T}{2} \frac{d\Delta^2(T)}{dT} \right] \frac{e^{\varepsilon/k_B T}}{(1 + e^{\varepsilon/k_B T})^2}$$

$$\zeta = \sqrt{\varepsilon^2 + \Delta^2(T)}$$

- Anisotropic S-wave or Non-monotonic d-wave

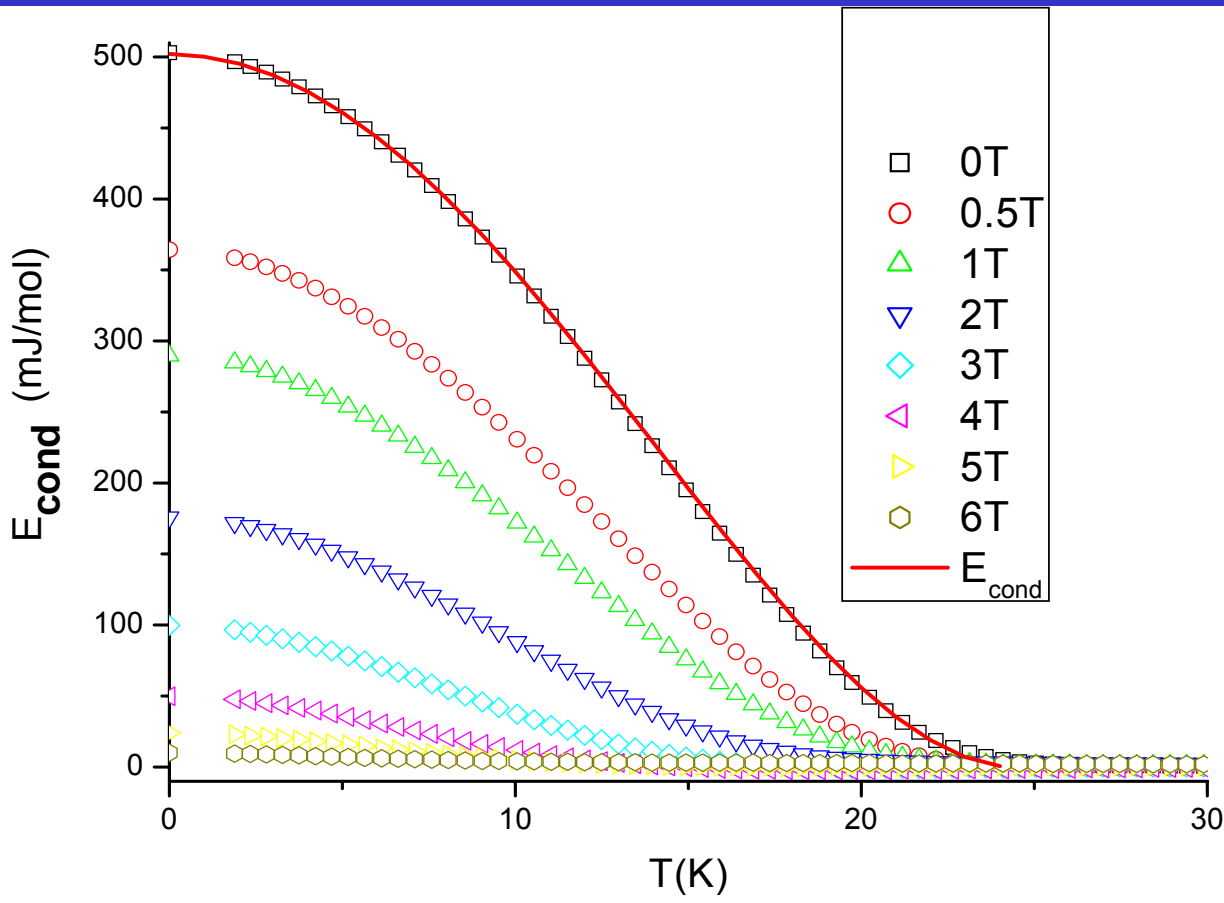


$$\Delta_d = \Delta_0 \cos 2\theta$$

$$\Delta_{nmd} = \Delta_0 [1.43 \cos 2\theta - 0.43 \cos 6\theta]$$

$$E_{cond} = H_c^2 / 8\pi$$

$$H_c(T) = H_c(0) \left[1 - (T / T_c)^2 \right]$$



BCS model:

$$E_{cond} \approx \alpha N(E_F) \Delta_0^2 / 2$$

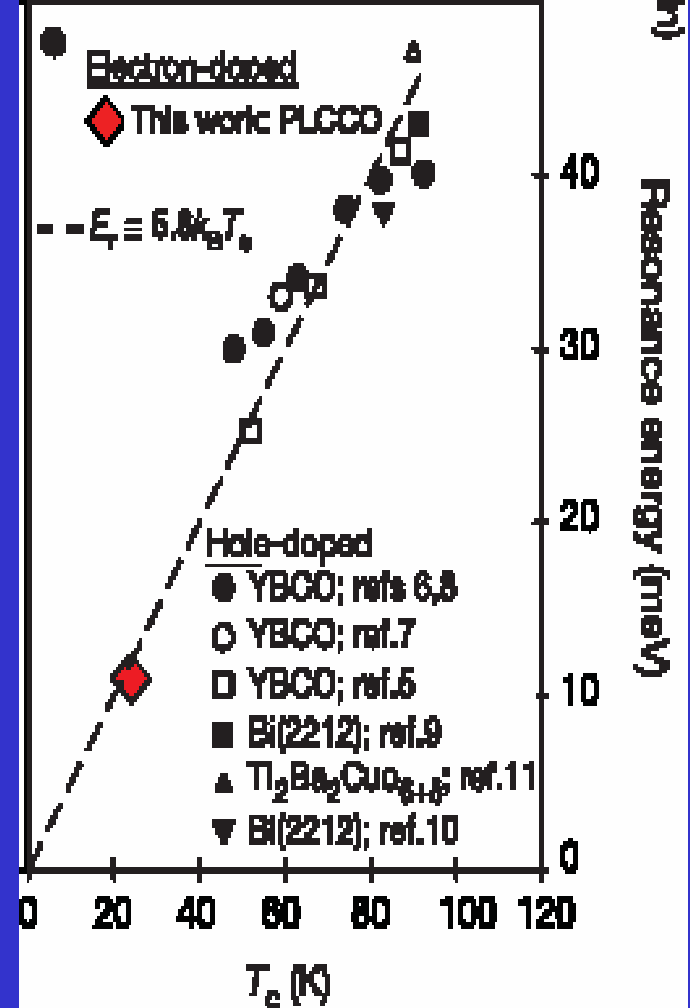
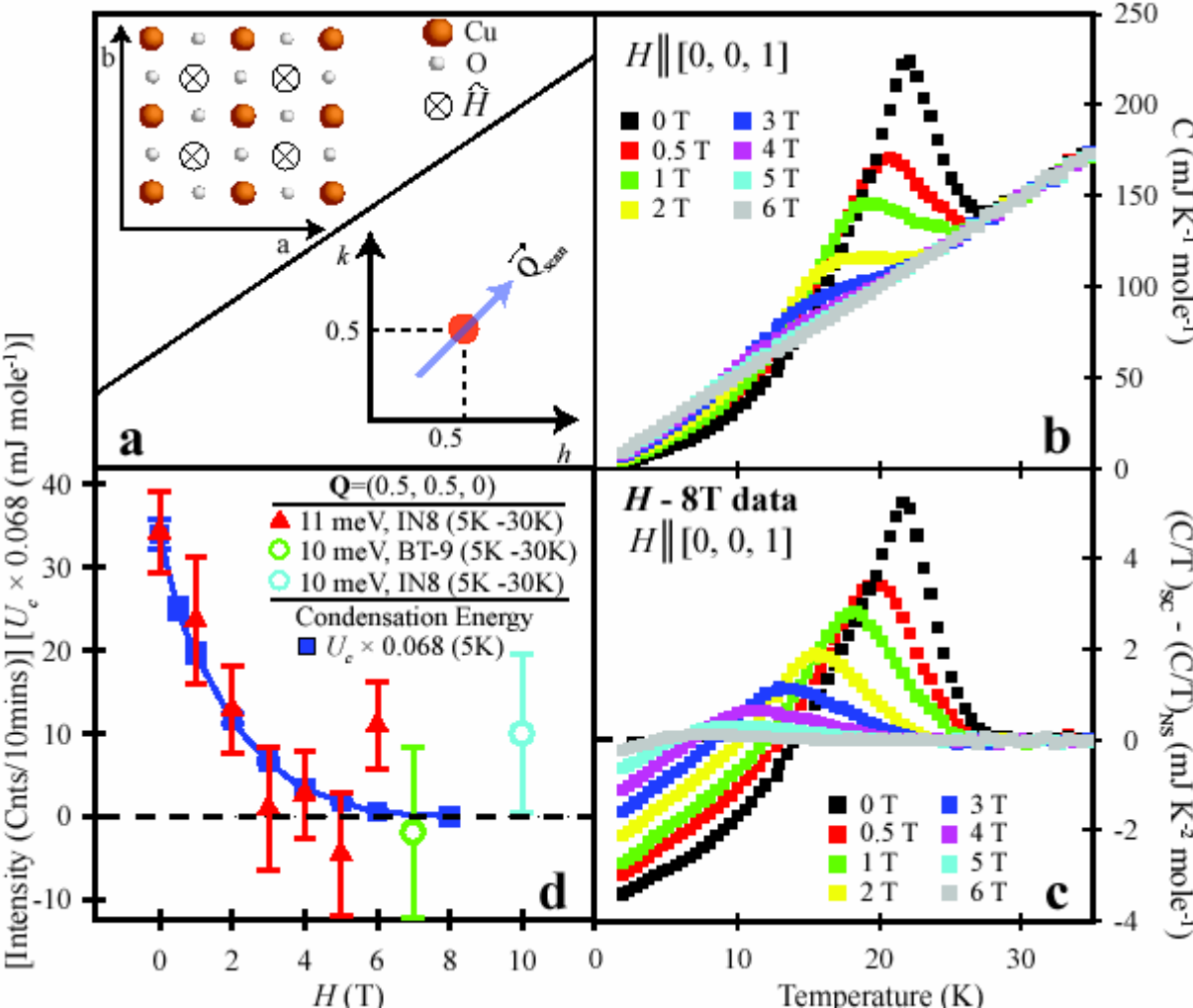
$$\approx \alpha \frac{3}{4\pi^2} \frac{1}{k_B^2} \gamma_n(0) \Delta_0^2$$

$$= 632 \text{ mJ} / \text{mol}$$

If taking $\alpha = 1$.

$E_{cond} = 502 \text{ mJ/mol}$. (experiment)

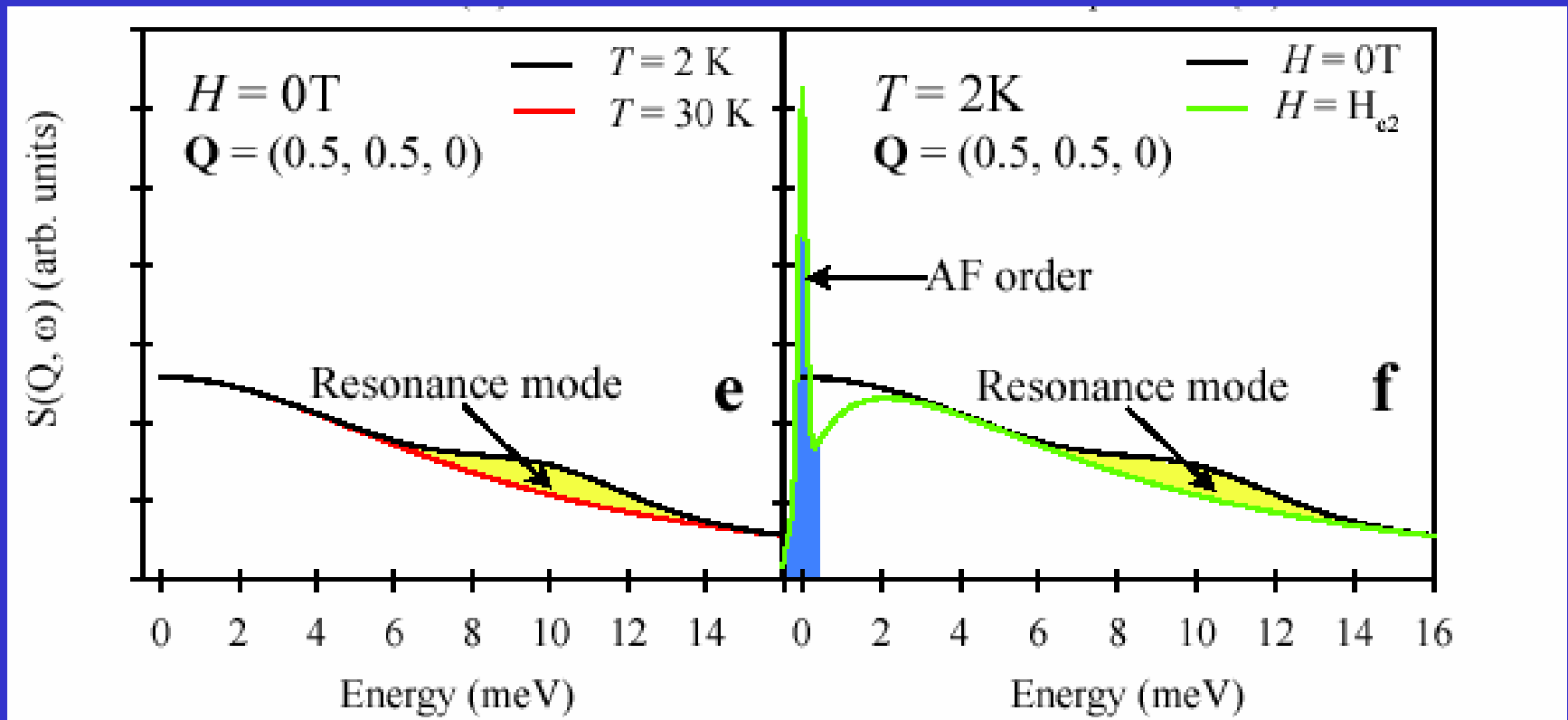
After the volume correction: $E_{cond} = 502-748 \text{ mJ/mol}$



Proc. National Acad. Sci.104, 15259 (2007).

Collaboration between P. C. Dai and H. H. Wen's group

S. Wilson, et al.,
Nature 442, 59 (2006).



Proc. National Acad. Sci. 104, 15259 (2007).

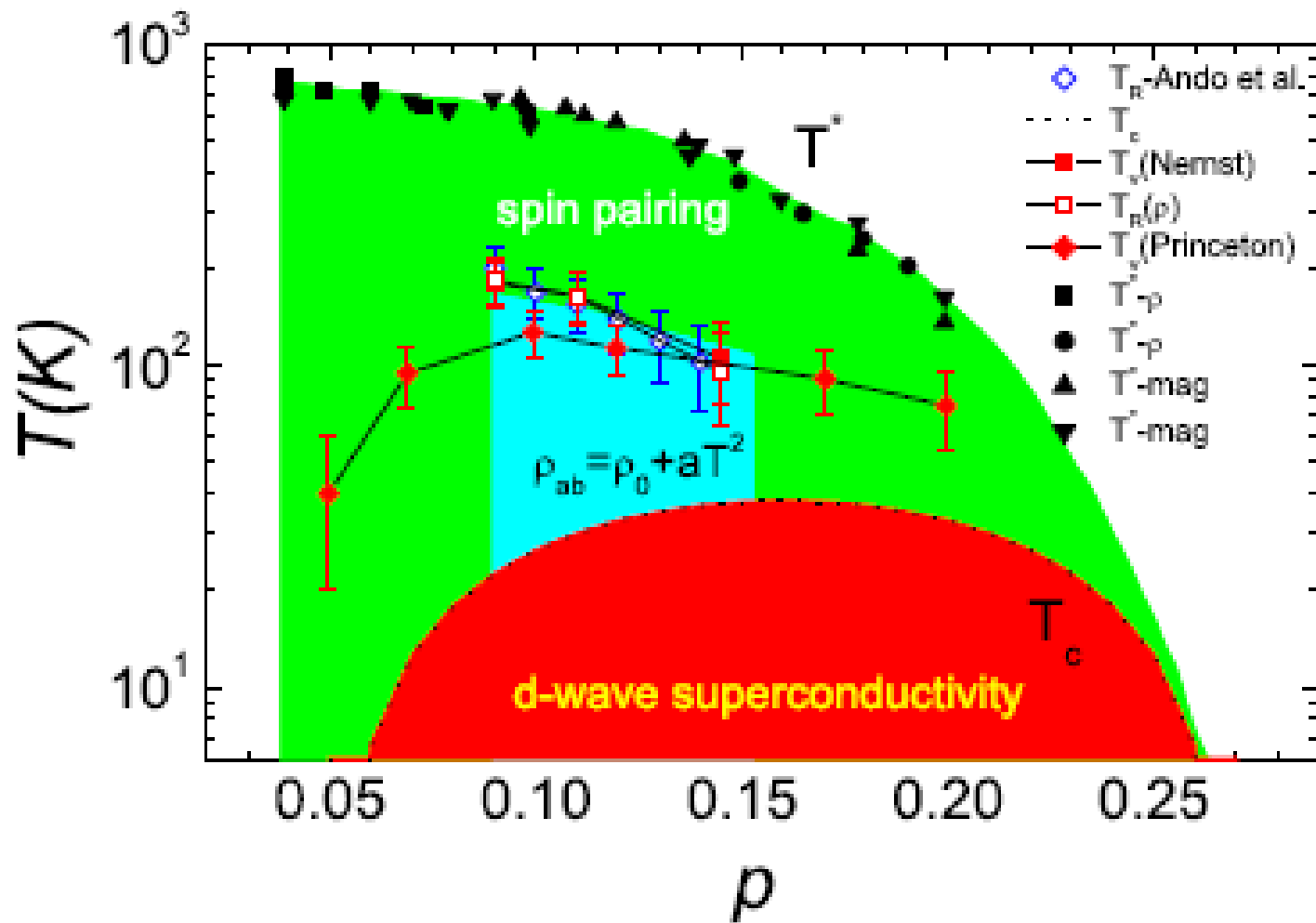
Collaboration between P. C. Dai and H. H. Wen's group

Summary from experiment

- In all doped regions, d-wave pairing symmetry holds;
- It seems to be BCS like in overdoped region although with some specialties: possible phase separation;
- The general quasiparticle gap becomes smaller towards more doping, which resembles the doping dependence of the AF correlation, or the RVB singlet pairing gap;
- Pseudogap phase has small Fermi surfaces, the superconductivity may be formed by the a new “gapping” on these small FS;
- There is a close relationship between the pseudogap and superconductivity.
- The resonance peak on the magnetic fluctuation spectrum has a close connection between the superconductivity condensation.

Phase diagram of cuprate superconductors

Based on the data of La-214



Possible pictures for HTS

1. Spin fluctuation mediated pairing (glue may or may not need)
2. RVB based picture: Mobil electrons swim in the Natural spin singlet-pairing background (glue does not need)
3. Electron-phonon pairing (glue need)

Thank you !